Physics 101 P

General Physics I

Problem Sessions – Week 3

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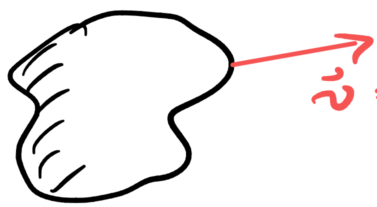
Newton's Laws of Motion

NI: An object at rest or traveling in uniform motion will remain at rest or traveling in uniform motion unless and until an external force is applied.

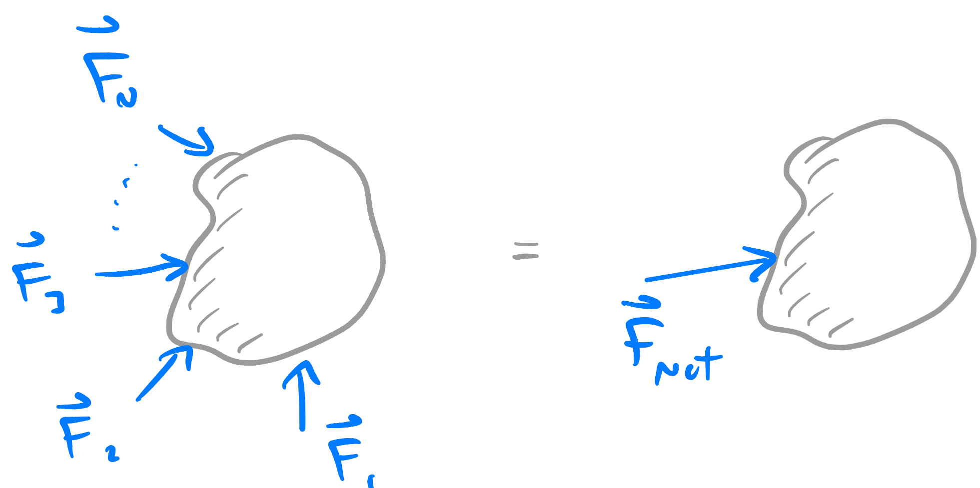


$\vec{v}$ = constant or zero

$$\vec{F}_{Net} = \sum_i \vec{F}_i = \vec{0} \Rightarrow \vec{v} = \text{constant}$$

($\vec{v} = \vec{0}$ is special case)

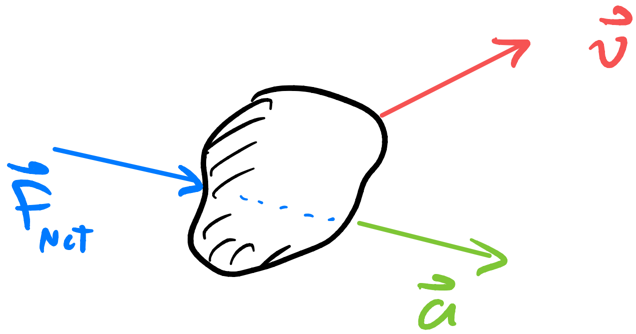
Note: $\vec{F}_{Net}$ is Net force on object



$$\vec{F}_{Net} = \sum_{i=0}^{N} \vec{F}_i \qquad \text{(superposition)}$$

$$= \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_N$$

NII: The acceleration of a body is directly proportional to the net force acting on it, in the direction of the applied force, and inversely proportional to the mass of the object.



$$\vec{F}_{Net} = \sum_i \vec{F}_i \neq \vec{0}$$

Net Force on object

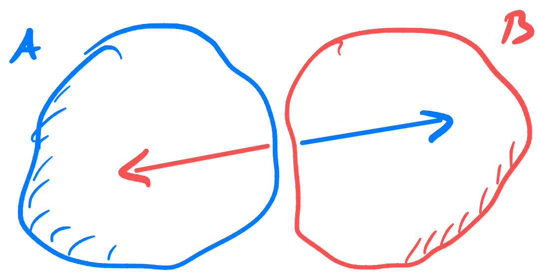
$$\vec{a} = \frac{\vec{F}_{Net}}{m}$$

acceleration of object "change in motion"

mass of object

$$\vec{a} = \frac{d\vec{v}}{dt}, \quad \vec{v} = \frac{d\vec{r}}{dt}$$

NIII : For every action there is
an equal and opposite reaction



$\vec{F}_{B \text{ on } A}$ $\vec{F}_{A \text{ on } B}$

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

Analysis with Newton's Laws

Solving problems with multiple forces/objects

1. Draw a simplified version of object.

   Need 1 Free-body Diagram (FBD) for each object.

2. Set up coordinate system

3. Identify all forces on object

   Do NOT include forces exerted BY object on other objects

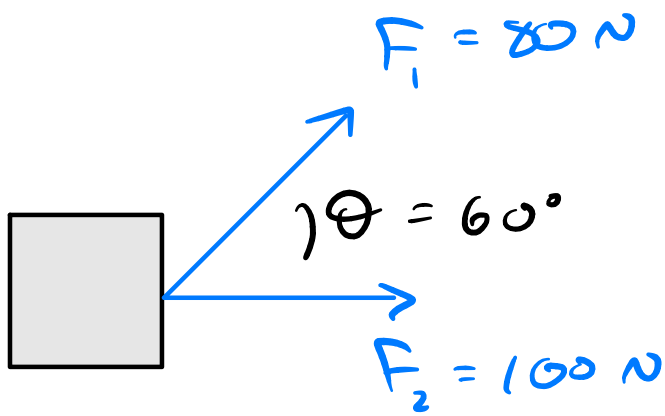
4. Draw vector arrows representing all forces on object

5. Find components of forces, sum to find resultant

6. Apply NII ⇒ Determine motion

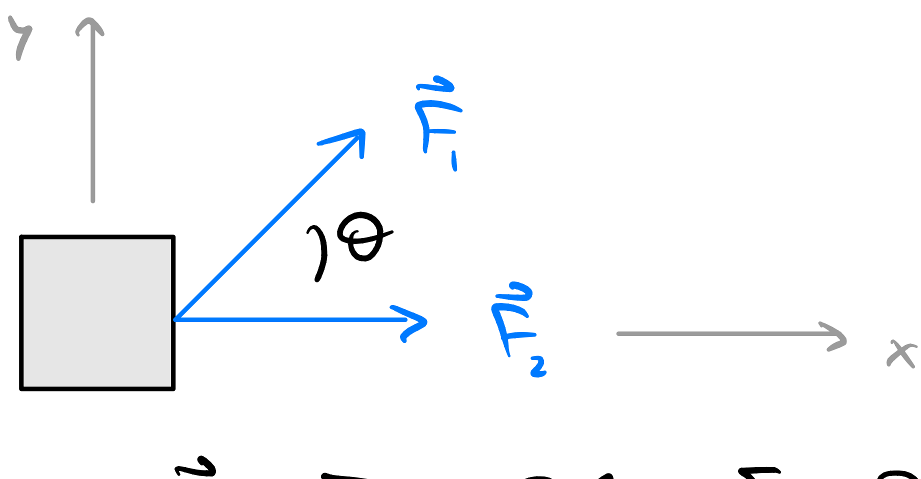
Example

Two forces, 80N and 100N acting at an angle of 60° with each other, pull on object. What single force (the resultant) would replace the two forces?



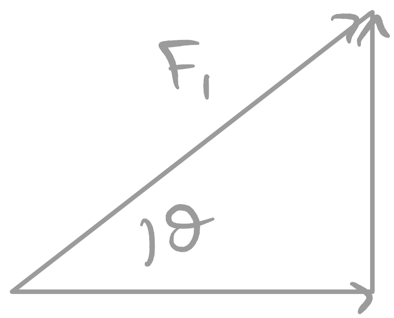
Solution

Set up a coordinate system



$$\vec{F}_1 = F_1 \cos\theta \,\hat{i} + F_1 \sin\theta \,\hat{j}$$

$$\vec{F}_2 = F_2 \,\hat{i}$$



$F_{1y} = F_1 \sin\theta$

$F_{1x} = F_1 \cos\theta$

$\cos 60^\circ = \frac{1}{2}$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

Total (Net) force

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$= (F_1 \cos\theta + F_2)\,\hat{i} + F_1 \sin\theta\,\hat{j}$$

$$= \left(\tfrac{1}{2} F_1 + F_2\right)\hat{i} + \frac{\sqrt{3}}{2} F_1\,\hat{j}$$

$$= 140\,N\,\hat{i} + 40\sqrt{3}\,N\,\hat{j}$$

$$\simeq 140\,N\,\hat{i} + 69\,N\,\hat{j}$$

$$\boxed{\vec{F} = 140\,N\,\hat{i} + 69\,N\,\hat{j}}$$

Magnitude of resultant force   26.329°

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(140\,N)^2 + (40\sqrt{3}\,N)^2}$$

$$\simeq 156\ N$$

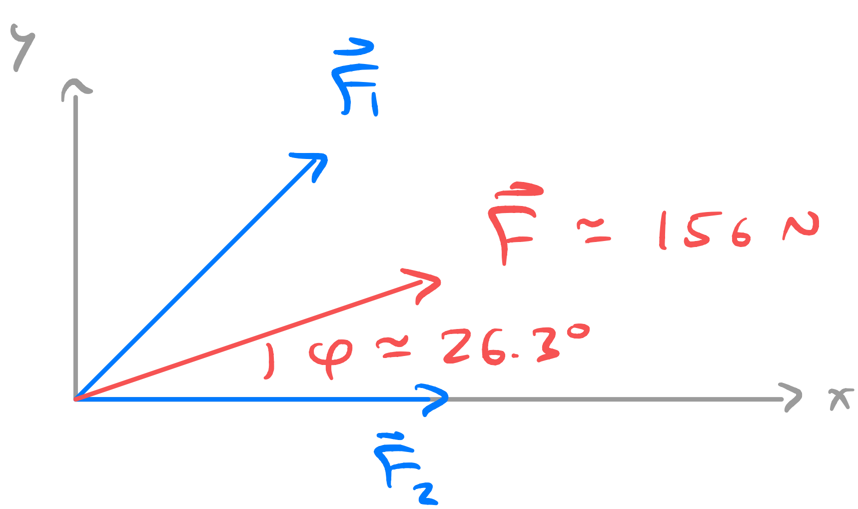
Angle w.r.t. x-axis (or $\vec{F}_2$)

$$\varphi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$= \tan^{-1}\left(\frac{40\sqrt{3}}{140}\right)$$

$$\simeq 26.3°$$

$$\boxed{\begin{aligned} F &= 156\,N \\ \varphi &= 26.3° \end{aligned}}$$



Example

A car whose weight is 3000 lbs is on a ramp which makes an angle $20°$ to the horizontal.

(a) What is the mass of the car in kg?

(b) How large a perpendicular force must the ramp withstand if it is not to break under the cars weight?

Solution

(a) Weight $W = mg$

Now, $1 \text{ lbs} \simeq 4.45 \text{ N}$

$$\Rightarrow W = 3000 \text{ lbs}$$

$$= 3000 \text{ lbs} \left(\frac{4.45 \text{ N}}{1 \text{ lbs}} \right)$$

$$= 13350 \text{ N}$$

$$\simeq 1.34 \times 10^4 \text{ N}$$

Now, $m = \frac{W}{g}$

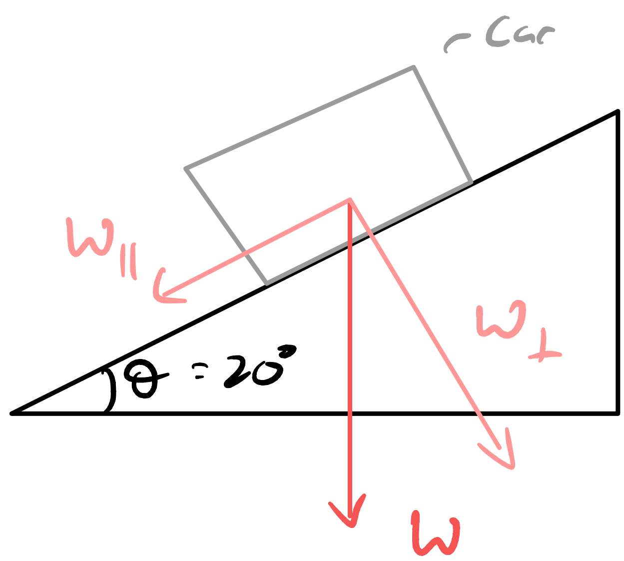
$$= \frac{1.34 \times 10^4 \ N}{9.8 \ m/s^2}$$

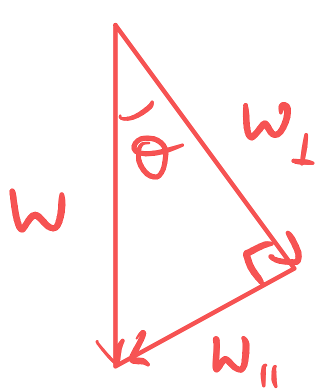
$$\simeq 1400 \ kg$$

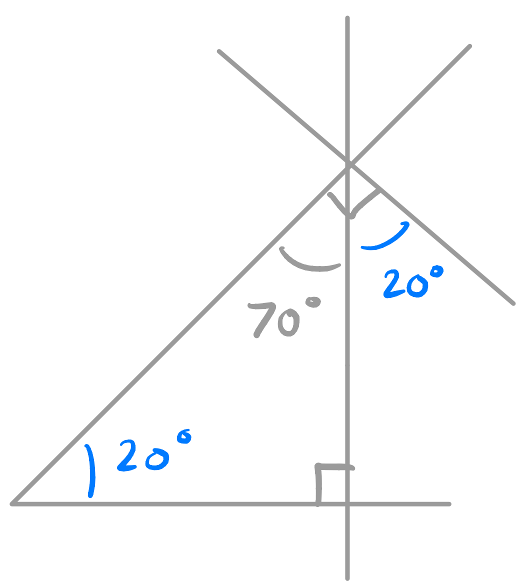
$$\Rightarrow \boxed{m = 1400 \ kg}$$

$1N = 1 \, kg \cdot \frac{m}{s^2}$

(b)







So,

$$W_{\perp} = W\cos\theta$$

$$= 3000\ \text{lbs}\ \cos(20^{\circ})$$

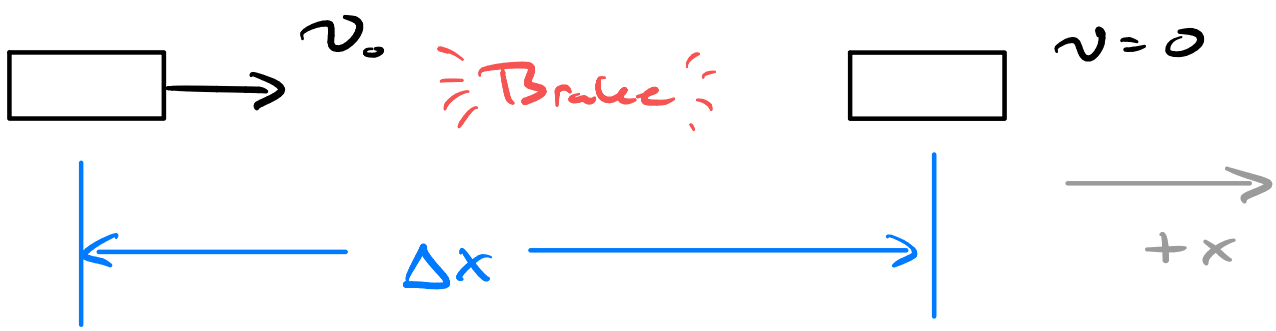
$$\simeq 2819\ \text{lbs}$$

$$\boxed{W_{\perp} = 2819\ \text{lbs}}$$

Example

What is the net force (in Newtons) required to stop a 1500 kg car moving with a speed of 55 mph within a distance of 200 ft (61 m)?

Solution



Want $\vec{F} = m\vec{a}$, what is a?

Recall $v^2 = v_0^2 + 2a\Delta x$

But, $v = 0 \Rightarrow a = \dfrac{-v_0^2}{2\Delta x}$

Now,

$$v_0 = 55\ \frac{mi}{h} \cdot \left(\frac{1609\ m}{1\ mi}\right) \cdot \left(\frac{1\ h}{3600\ s}\right)$$

$$= 24.6\ m/s$$

So,

$$a = -\frac{v_0^2}{2\Delta x}$$

$$= -\frac{(24.6\ m/s)^2}{2(61\ m)}$$

$$= -4.96\ m/s^2$$

Therefore, force is

$$F = ma$$

$$= (1500\ kg)(-4.96\ m/s^2)$$

$$= -7440\ N$$

Force is negative, why? Force goes against motion, which is $+x$ in our coordinate system.

$$\boxed{F = -7440\ N}$$

N.B. $v^2 = v_0^2 + 2a\Delta x$, derivation

$$a = \frac{dv}{dt} \quad , \text{ so } \quad \frac{dx}{dt} = v$$

$$\Rightarrow \quad v \cdot a = v \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dx}{dt} \cdot a = v \cdot \frac{dv}{dt} = \frac{1}{2} \frac{dv^2}{dt}$$

a = constant, $\quad a \frac{dx}{dt} = \frac{d}{dt}(a x)$

So, $$\frac{d(ax)}{dt} = \frac{d}{dt}\left(\frac{v^3}{2} \right)$$

Integrate $\Rightarrow \quad a \int_{x_0}^{x} dx = \frac{1}{2} \int_{v_0^2}^{v^2} dv^2$

$$\Rightarrow \quad \underbrace{a(x - x_0)}_{\Delta x} = \frac{1}{2} \left(v^2 - v_0^2 \right)$$

$$\Rightarrow \quad v^2 = v_0^2 + 2a \Delta x \qquad \blacksquare$$

Example

A friend has given you a special gift, a box of mass 10.0 kg with a mystery surprise inside. The box is resting on a smooth (frictionless) horizontal surface of a table.

(a) Determine the weight of the box and the normal force exerted on it by the table.

(b) Now your friend pushes down on the box with a force of 40.0 N. Determine the normal force exerted on the box by the table.

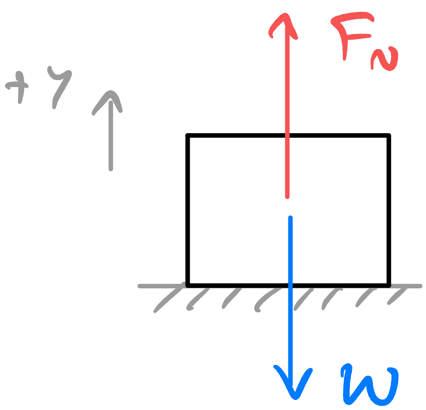
(c) If your friend pulls upward on the box with a force of 40.0 N, what now is the normal force exerted on the box by the table?

(d) What would happen if a person pulls upward on the box with a force of 100.0 N?

Solution

(a) $W = mg = (10.0 \text{ kg})(9.8 \text{ m/s}^2)$

$= 98 \text{ N}$ $\blacksquare$

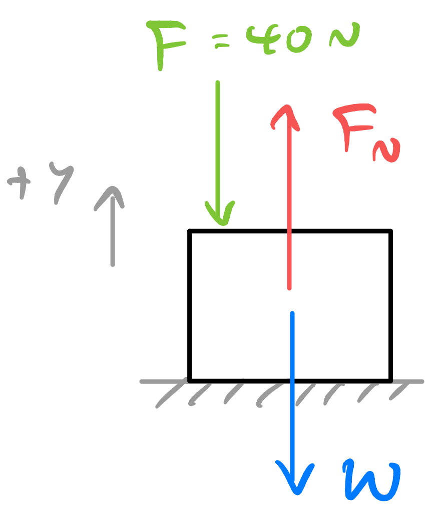


$$\frac{\sum F = ma}{F_N - W = 0}$$

$\Rightarrow F_N = W$

$= 98 \text{ N}$ $\blacksquare$

(b)

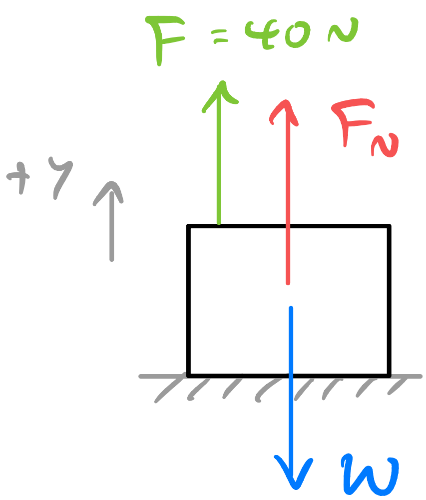


$$\frac{\sum F = ma}{F_N - W - F = 0}$$

$\Rightarrow F_N = F + W$

$= 138 \text{ N}$ $\blacksquare$

(c)



$$\sum F = ma$$

$$F_N - W + F = 0$$

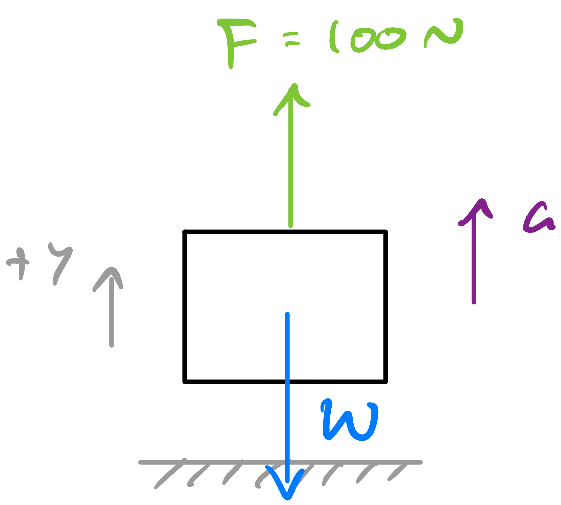
$$\Rightarrow F_N = W - F$$

$$= 58 \text{ N} \blacksquare$$

(d) if $F = 100$ N, $F > W$

$\Rightarrow$ Box will be lifted

$\Rightarrow$ No contact with table!



$$\sum F = ma$$

$$F - W = ma$$

$$\Rightarrow a = \frac{F - W}{m}$$

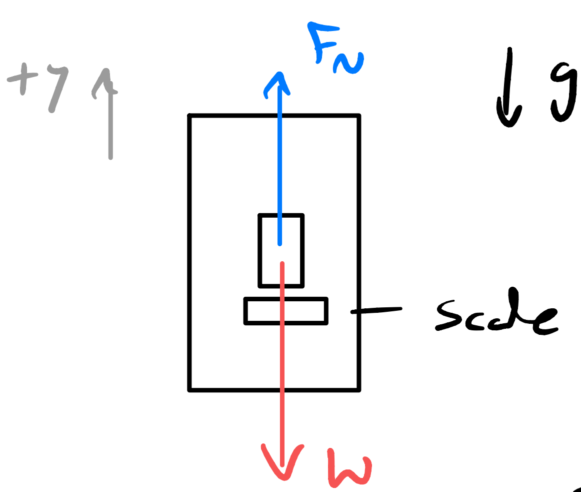
$$= \frac{100 \text{ N} - 98 \text{ N}}{10 \text{ kg}}$$

$$= 0.2 \text{ m/s} \blacksquare$$

Example

You are standing on a bathroom scale in an elevator. Suddenly, the elevator cable is cut & the safety devices fail, so that the elevator is in free fall. In your final moments, you look at the scale and see your weight. What would you read off the bathroom scale?

Solution



$$\sum F = ma$$

$$F_N - W = m(-g)$$

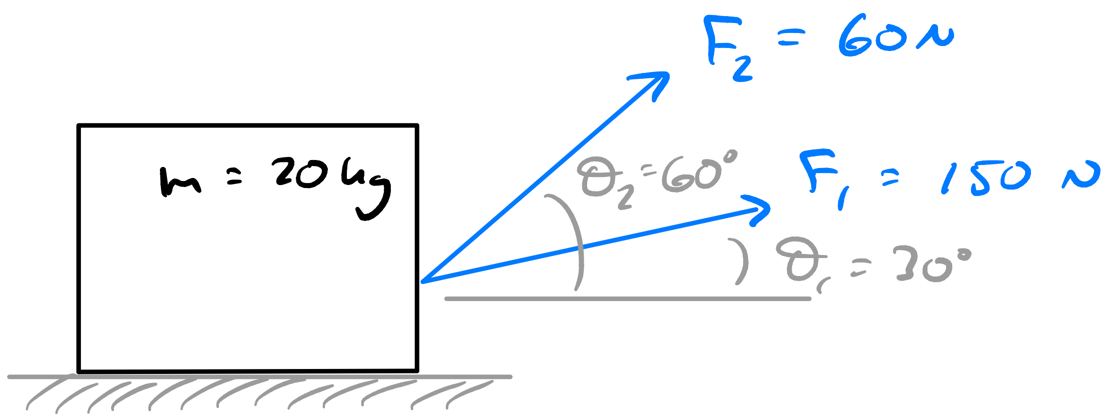
$$\Rightarrow F_N = W - mg$$

But, $W = mg$

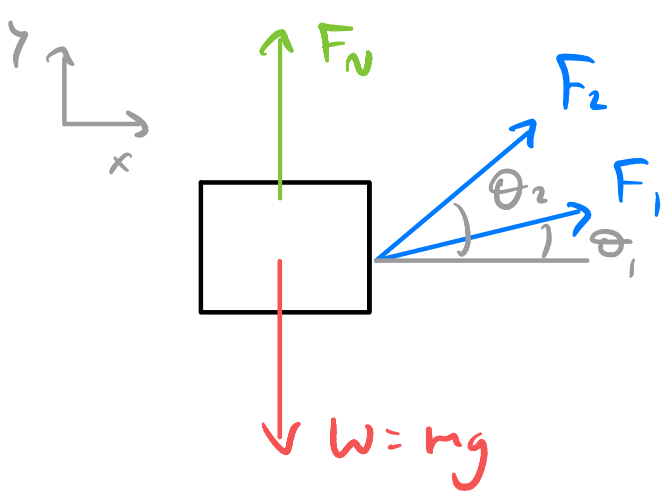
$$\Rightarrow F_N = mg - mg$$
$$= 0 \quad \blacksquare$$

Example

Two kids are pulling a 20 kg box on a frictionless horizontal floor, as shown. One kid pulls with a force of 150 N at 30° above horizontal, while the other child pulls with a force 60 N at 45° from horizontal. Find the horizontal acceleration and normal force of the box.



Solution



$$\begin{cases} F_{1x} = F_1 \cos\theta_1 \\ F_{1y} = F_1 \sin\theta_1 \end{cases}$$

$$\begin{cases} F_{2x} = F_2 \cos\theta_2 \\ F_{2y} = F_2 \sin\theta_2 \end{cases}$$

$$\sum \vec{F} = m\vec{a}$$

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x: $F_1 \cos\theta_1 + F_2 \cos\theta_2 = m\overset{?}{a}$

y: $F_1 \sin\theta_1 + F_2 \sin\theta_2 + \overset{?}{F_N} - mg = 0$

Two equations, two unknowns (a, F_N)

$$\Rightarrow \quad a = \frac{F_1 \cos\theta_1 + F_2 \cos\theta_2}{m}$$

$$= \frac{150\text{ N} \cos 30^\circ + 60\text{ N} \cos 45^\circ}{20\text{ kg}}$$

$$= 8.6 \text{ m/s}^2 \quad \blacksquare$$

$$\Rightarrow \quad F_N = mg - F_1 \sin\theta_1 - F_2 \sin\theta_2$$

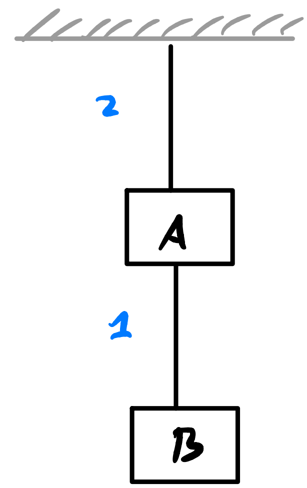
$$= (20\text{ kg})(9.8\text{ m/s}^2) - 150\text{ N} \sin 30^\circ - 60\text{ N} \sin 45^\circ$$

$$= 78.6 \text{ N} \quad \blacksquare$$

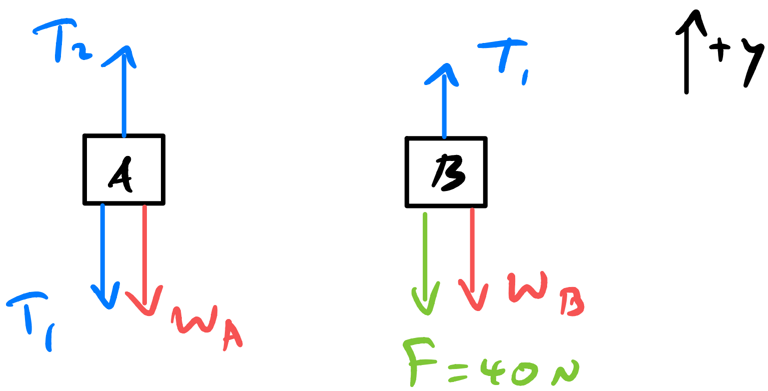
Example

Two boxes, A and B, are connected by a lightweight cord and are hanging from the ceiling. The boxes have masses of $m_A = 12.0\ \text{kg}$ & $m_B = 10.0\ \text{kg}$. You pull the lower box (box B) with a force of 40 N. Find the tension in the two pieces of rope.

Solution



Free body diagrams



$$\sum F = ma$$

$$A: \quad T_2 - T_1 - w_A = 0 \qquad (1) \quad ; \quad w_A = m_A g$$

$$B: \quad T_1 - w_B - F = 0 \qquad (2) \quad ; \quad w_B = m_B g$$

By (2),
$$T_1 = F + w_B$$
$$= 40\ \text{N} + (10\ \text{kg})(9.8\ \text{m s}^{-2})$$
$$= 138\ \text{N} \quad \blacksquare$$

from (1),

$$T_2 = T_1 + W_A$$

$$= (m_A + m_B)g + F$$

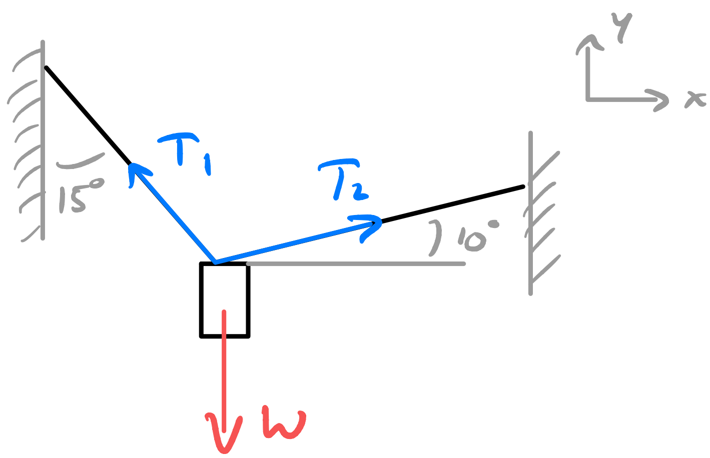
$$= (12\,\text{kg} + 10\,\text{kg})(9.8\,\text{m/s}^2) + 40\,\text{N}$$

$$\simeq 256\,\text{N} \quad \blacksquare$$

Example

After a mishap, a 76 kg circus performer clings to a trapeze, which is being pulled to the side by another circus artist, as shown. Calculate the tension in the two ropes if the person is momentarily motionless.

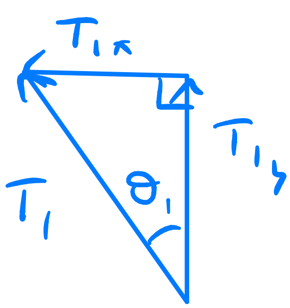
Solution



Let $\theta_1 = 15°$

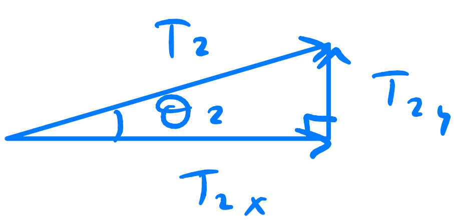
$\theta_2 = 10°$

Resolve T_1 & T_2



$$T_{1x} = T_1 \sin\theta_1$$

$$T_{1y} = T_1 \cos\theta_1$$



$$T_{2x} = T_2 \cos\theta_2$$

$$T_{2y} = T_2 \sin\theta_2$$

$$\sum \vec{F} = m\vec{a}$$

$$x: \quad T_2 \cos\theta_2 - T_1 \sin\theta_1 = 0 \qquad (1)$$

$$y: \quad T_2 \sin\theta_2 + T_1 \cos\theta_1 - W = 0 \qquad (2)$$

and $W = mg$

From (1), $T_2 = T_1 \dfrac{\sin\theta_1}{\cos\theta_2}$

From (2),

$$T_1 \frac{\sin\theta_1}{\cos\theta_2} \sin\theta_2 + T_1 \cos\theta_1 = mg$$

$$\Rightarrow \quad T_1 = \frac{mg}{\cos\theta_1 + \sin\theta_1 \tan\theta_2}$$

$$= 736.3 \text{ N} \quad \blacksquare$$

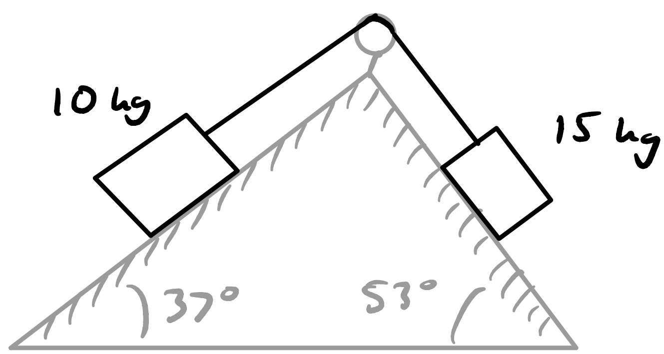
Back to (1)

$$T_2 = T_1 \frac{\sin\theta_1}{\cos\theta_2}$$

$$= 193.5 \text{ N} \quad \blacksquare$$

Example

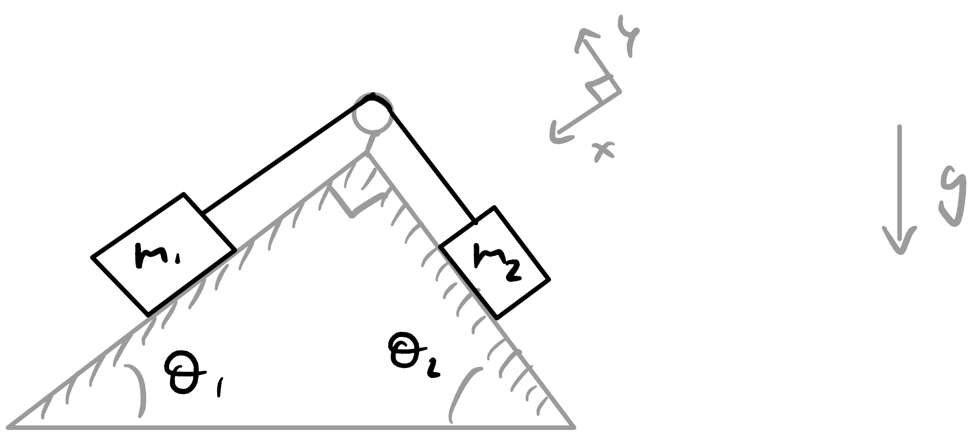
Two carts are connected by a cord that passes over a small frictionless pulley. Each cart rolls freely with negligible friction. Calculate the acceleration of the carts and the tension in the cord.



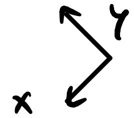
Solution

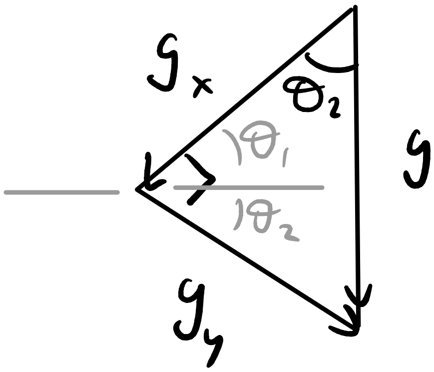
Note that $37° + 53° = 90°$

$\Rightarrow$ Can choose convenient coordinate system



Now,



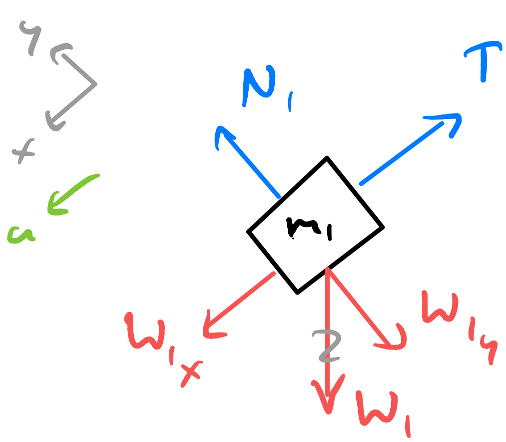


$$g_x = g\cos\theta_2$$

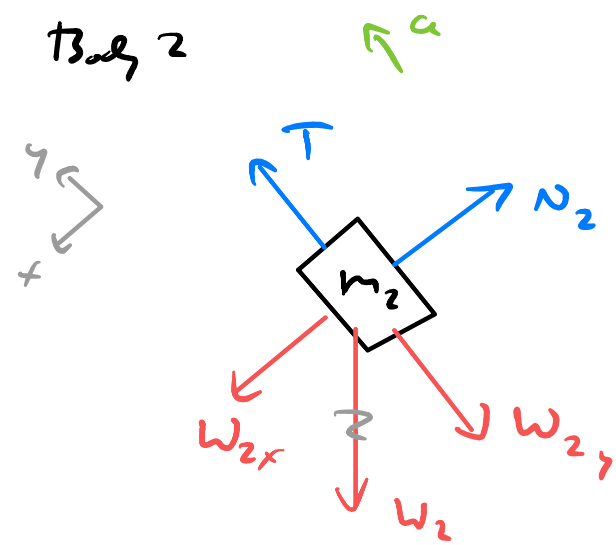
$$g_y = g\sin\theta_2$$

Now, FBD's

Body 1



Body 2



$$\sum \vec{F} = m\vec{a}$$

Assume direction of "a": if negative, goes other way

① x: $W_{1x} - T = m_1 a$ (1)

y: $N_1 - W_{1y} = 0$ (2)

② x: $W_{2x} - N_2 = 0$ (3)

y: $T - W_{2y} = m_2 a$ (4)

$$W_{1x} - T = m_1 a \quad (1) \quad \Rightarrow \quad m_1 g \cos\theta_2 - T = m_1 a$$

$$N_1 - W_{1y} = 0 \quad (2) \quad \Rightarrow \quad N_1 - m_1 g \sin\theta_2 = 0$$

$$W_{2x} - N_2 = 0 \quad (3) \quad \Rightarrow \quad m_2 g \cos\theta_2 - N_2 = 0$$

$$T - W_{2y} = m_2 a \quad (4) \quad \Rightarrow \quad T - m_2 g \sin\theta_2 = m_2 a$$

Combine (1) & (2), solve for "a"

$$m_1 g \cos\theta_2 - m_2 a - m_2 g \sin\theta_2 = m_1 a$$

$$\Rightarrow \quad a = \frac{g}{m_1 + m_2} \left(m_1 \cos\theta_2 - m_2 \sin\theta_2 \right)$$

$$\simeq -2.3 \ m/s^2 \quad \blacksquare$$

Sub "a" into (1)

$$T = m_1 g \cos\theta_2 - m_1 a$$

$$\simeq 82.3 \ N \quad \blacksquare$$

N.B. $\cos(\theta_2) = \cos\left(\frac{\pi}{2} - \theta_1\right) = \sin\theta_1$

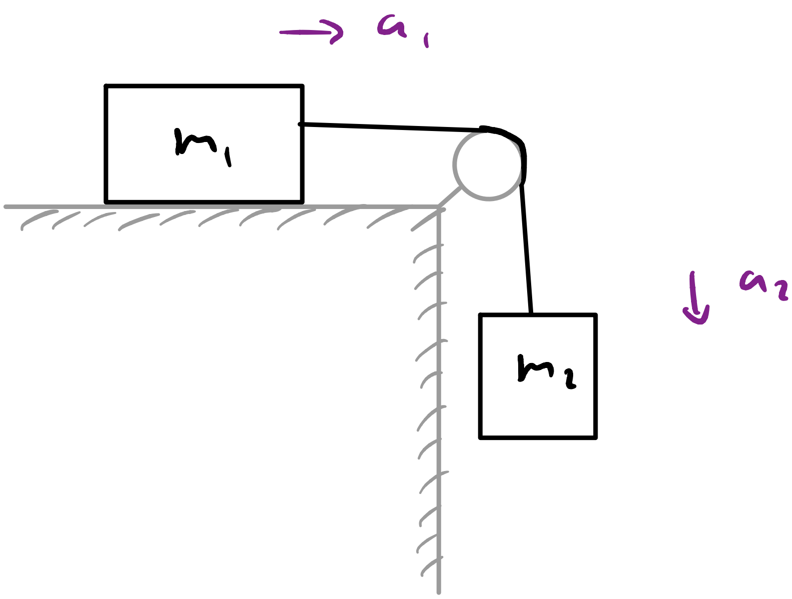
$$\Rightarrow \quad a = \frac{g}{m_1 + m_2} \left(m_1 \sin\theta_1 - m_2 \sin\theta_2 \right)$$

&

$$T = m_1 g \sin\theta_1 - m_1 a$$

Example

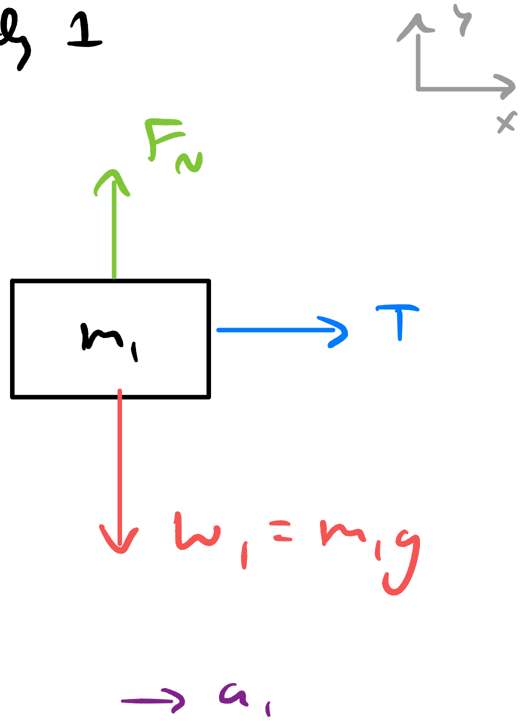
Two blocks are connected by a massless rope as shown. The mass of the block on the table is 4.0 kg and the hanging mass is 1.0 kg. The table and the pulley are frictionless. (a) Find the acceleration of the system. (b) Find the tension in the rope. (c) Find the speed with which the hanging mass hits the floor if it starts from rest and is initially located 1.0 m from the floor.



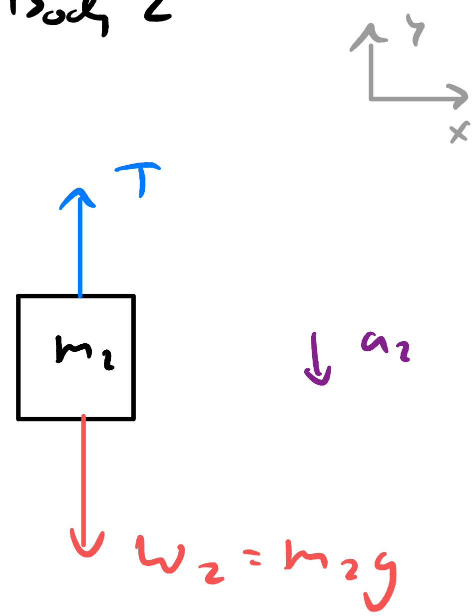
Solution

Free Body Diagrams

Body 1



Body 2



Now, rope is inextensible, $\Rightarrow a_1 = a_2 \equiv a$

① $x: \quad T = m_1 a$

$y: \quad F_N - w_1 = 0$

② $x: \quad 0 = 0$

$y: \quad T - w_2 = -m_2 a$

$\therefore$

$$T = m_1 a \quad (1)$$

$$F_N = m_1 g \quad (2)$$

$$T - m_2 g = -m_2 a \quad (3)$$

put (1) into (3)

$$m_1 a - m_2 g = -m_2 a$$

$$\Rightarrow \quad a = \frac{m_2 g}{m_1 + m_2}$$

$$= 1.96 \ \mathrm{m/s^2} \qquad \blacksquare$$

Tension from a,

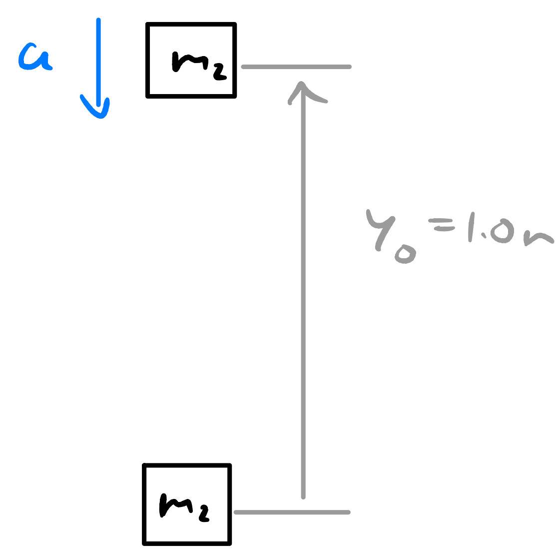
$$T = m_1 a$$

$$= \frac{m_1 m_2 g}{m_1 + m_2}$$

$$= 7.84 \ \mathrm{N} \qquad \blacksquare$$

If $\quad y_0 = 1.0\ \mathrm{m}, \quad y = 0$

$\quad v_0 = 0, \quad v = ?$



$$v^2 = -2a(y - y_0)$$

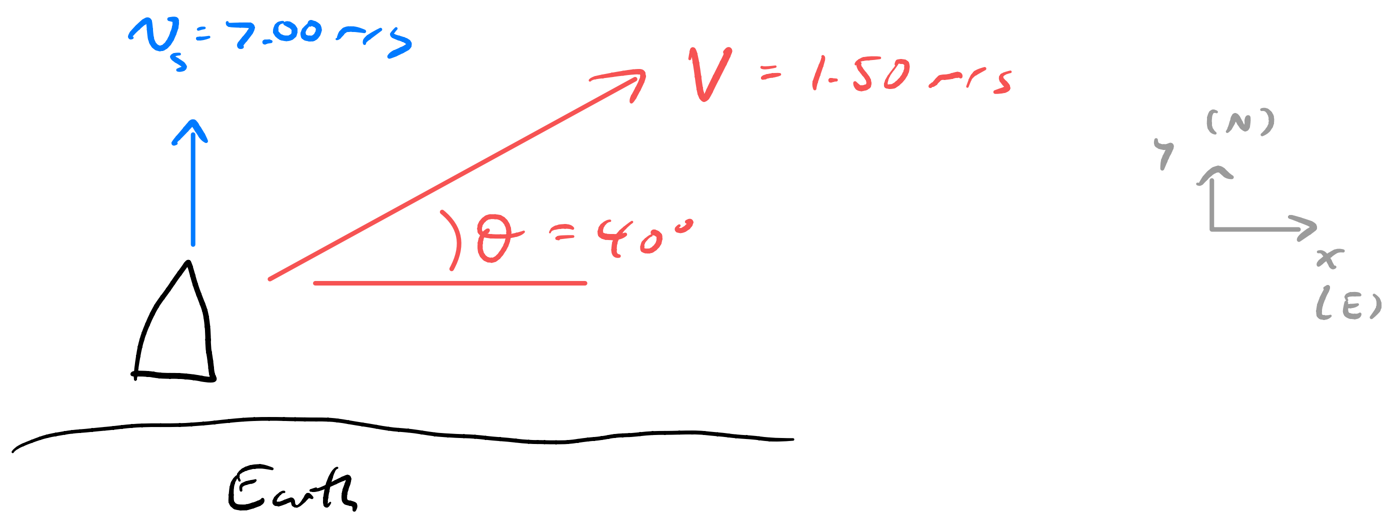
$$= \frac{2 m_2 g}{m_1 + m_2} y_0$$

$$\Rightarrow v = 1.98 \ \mathrm{m/s} \qquad \blacksquare$$

Example

A ship sets sail from Rotterdam, heading due north at 7.00 m/s relative to the water. The local ocean current is 1.50 m/s in a direction 40.0° north of east. What is the velocity of the ship relative to Earth?

Solution



$$\vec{v}_{s/w} + \vec{V}_{w/E} = \vec{v}_{s/E}$$

ship velocity wrt water

ocean current wrt Earth

velocity of ship wrt Earth

Now,

$$\vec{V}_{w/E} = 1.50\,\text{m/s}\cos 40^\circ\,\hat{i} + 1.50\,\text{m/s}\sin 40^\circ\,\hat{j}$$

$$= 1.15\,\text{m/s}\,\hat{i} + 0.96\,\text{m/s}\,\hat{j}$$

So,

$$\vec{v}_{s/E} = \vec{v}_{s/w} + \vec{V}_{w/E}$$

$$= (7.00\,\text{m/s}\,\hat{j}) + (1.15\,\text{m/s}\,\hat{i} + 0.96\,\text{m/s}\,\hat{j})$$

$$= 1.15\,\text{m/s}\,\hat{i} + 7.96\,\text{m/s}\,\hat{j}$$

$$v_{s/E} = \sqrt{v_{s/E_x}^2 + v_{s/E_y}^2}$$

$$\simeq 8.04\,\text{m/s}$$

$$\theta_{s/E} = \tan^{-1}\left(\frac{v_{s/E_y}}{v_{s/E_x}}\right)$$

$$= 81.8^\circ$$