Physics 101 P

General Physics I

Problem Sessions - Week 6

A.W. Jackura —— William & Mary

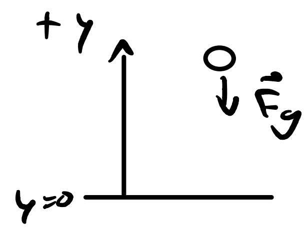
Work & Energy

$$W_{A\to B} = \int_{\substack{\text{path}\\ A\to B}} \vec{F}\cdot d\vec{r}$$

Work-Energy theorem: $W_{A\to B} = \Delta K$
$= K_B - K_A$

w/ Kinetic energy $K = \frac{1}{2}mv^2$

For near-Earth gravity, $\vec{F}_g = -mg\hat{j}$



Can define Potential Energy

$$W_{A\to B,g} = \int_A^B \vec{F}_g \cdot d\vec{r}$$

$$= -\Delta U$$

$$= -(U_B - U_A)$$

w/ $U = mgy$

Separate out gravity:

$$W_{fr_{A\to B}} = \underbrace{(K_B + U_B)}_{\equiv E_B} - \underbrace{(K_A + U_A)}_{\equiv E_A}$$

Total Mechanical energy

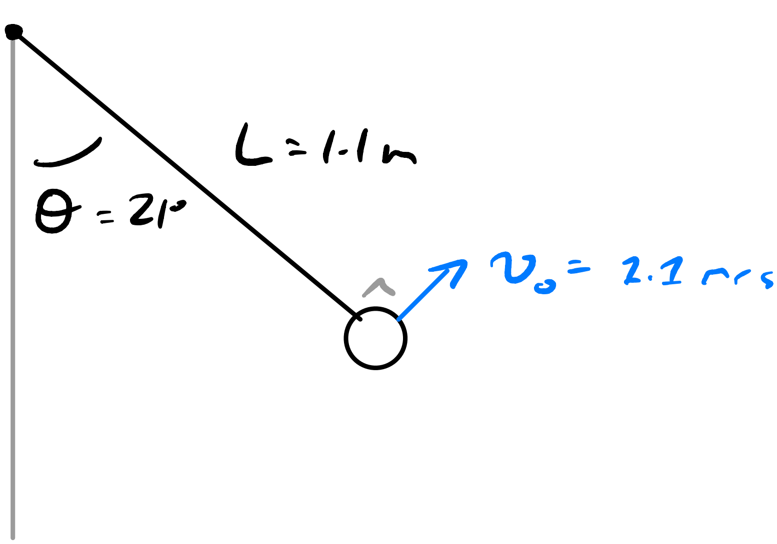
$$\boxed{\begin{array}{c}\text{If No Friction}\\ \hline E_B = E_A\end{array}}$$

Example

A ball is suspended from a light 1.1 m string. The string makes an angle of 21° with the vertical. The ball is then kicked up and to the right such that the string remains taut the entire time the ball swings upwards. This kick gives the ball an initial velocity of 1.1 m/s.

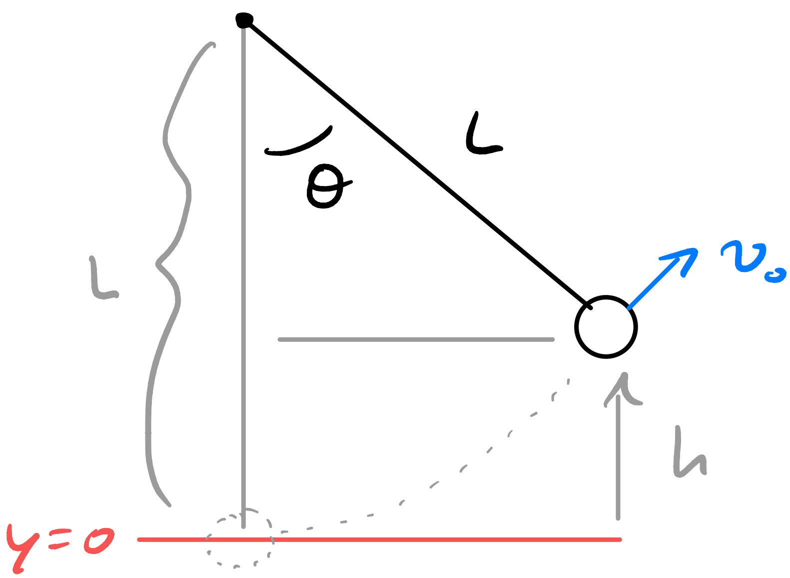
(a) What will be the speed of the ball when it reaches its lowest point?

(b) What will be the maximum angle the string makes with the vertical?



Solution

(a) Let $y=0$ be when the ball is at its lowest point.



No friction, total energy is conserved.

$$E_i = E_f$$

Final energy @ lowest point $y=0$

$$E_f = K_f + U_f$$

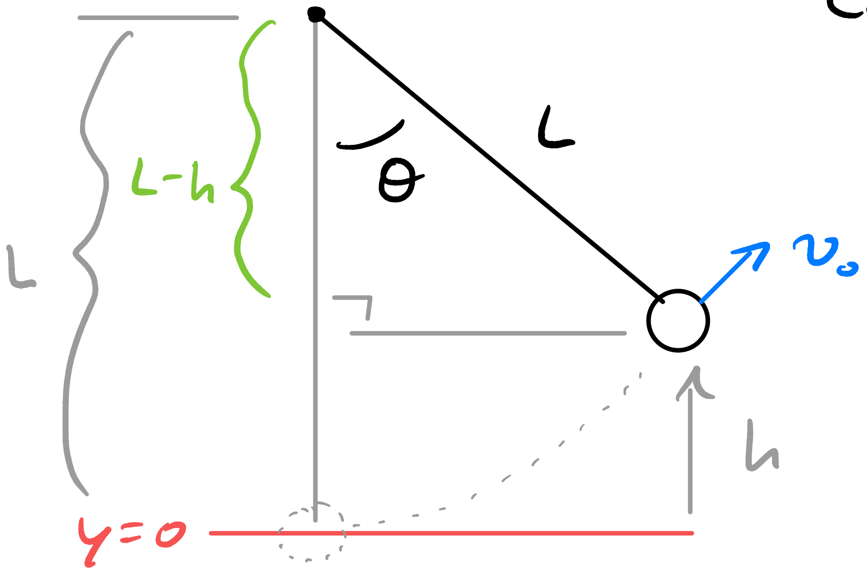
$$U_f = 0 \,, \quad K_f = \frac{1}{2} m v^2$$

Initial Energy @ initial height $y = h$

$$E_i = K_i + U_i$$

$$U_i = mgh \,, \quad K_i = \frac{1}{2} m v_0^2$$

Now, need h,



$$\cos\theta = \frac{L-h}{L}$$

$$\Rightarrow h = L(1-\cos\theta)$$

So, $U_i = mgh$

$$= mgL(1-\cos\theta)$$

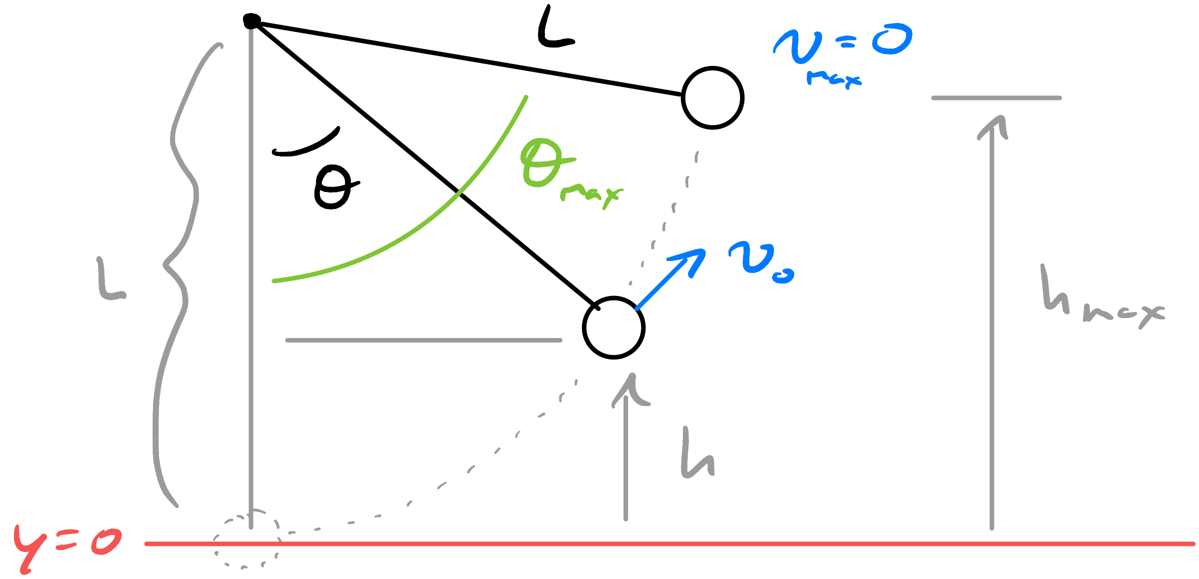
Therefore, $E_i = E_f$

$$\Rightarrow \quad \frac{1}{2}mv_0^2 + mgL(1-\cos\theta) = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{v_0^2 + 2gL(1-\cos\theta)}$$

$$\simeq 1.625 \text{ m/s} \quad \blacksquare$$

(b) To get maximum angle, θ_{max}, use Cons. of Energy!



From before, $h_{max} = L(1-\cos\theta_{max})$

$$h = L(1-\cos\theta)$$

So, $E_i = E_f$

Initial	Final
$U_i = mgh$	$U_f = mgh_{max}$
$K_i = \frac{1}{2}mv_0^2$	$K_f = 0$

$$\Rightarrow \frac{1}{2}mv_0^2 + mgh = mgh_{max}$$

$$\frac{1}{2} m v_0^2 + mgh = mgh_{max}$$

$$\Rightarrow \quad \frac{1}{2} v_0^2 + gL(\cancel{1} - \cos\theta) = gL(\cancel{1} - \cos\theta_{max})$$

$$\Rightarrow \quad \frac{1}{2} v_0^2 - gL\cos\theta = -gL\cos\theta_{max}$$

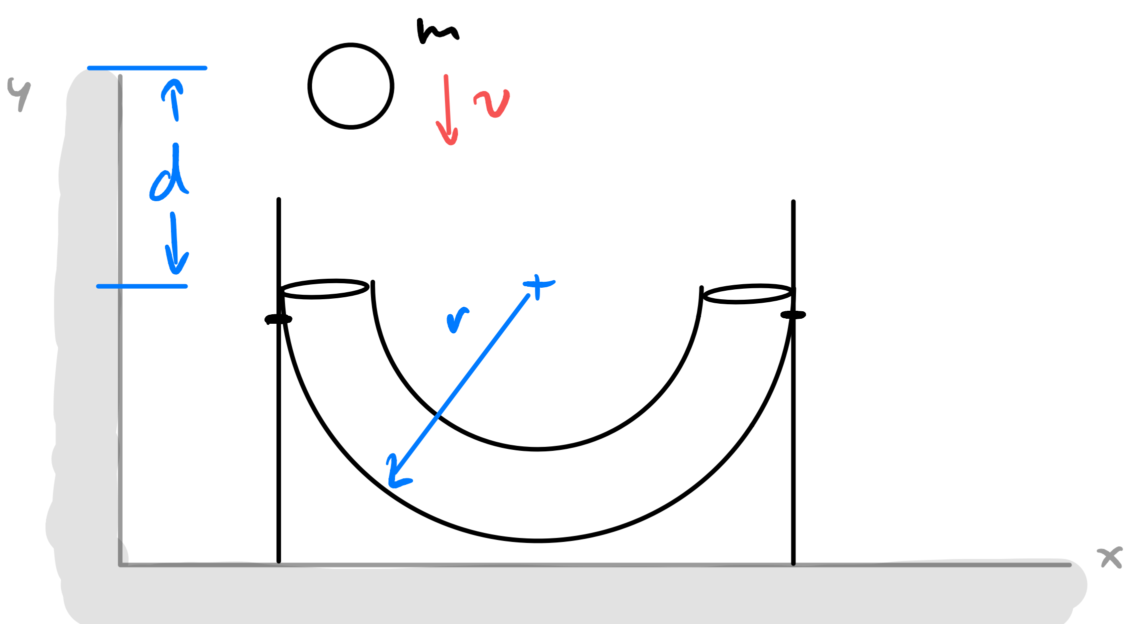
$$\Rightarrow \quad \cos\theta_{max} = \cos\theta - \frac{v_0^2}{2gL}$$

$$\Rightarrow \quad \theta_{max} = \cos^{-1}\left(\cos\theta - \frac{v_0^2}{2gL}\right)$$

$$\simeq 28.66^\circ \quad \blacksquare$$

Example

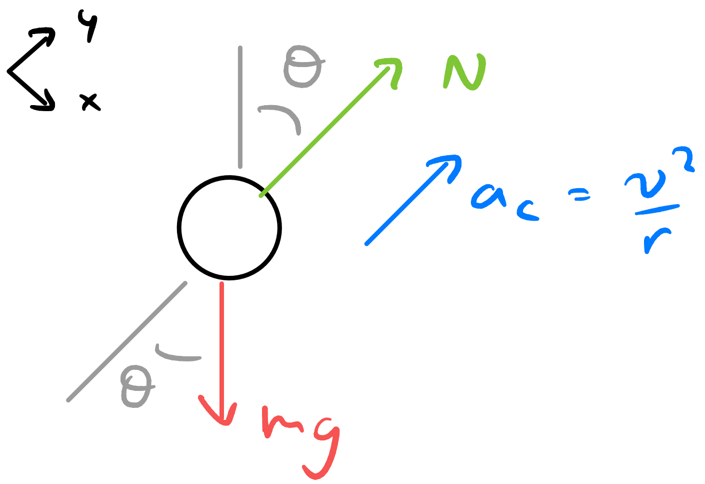
A bowling ball $m = 1.1$ kg is dropped from a height h. It falls a distance, d, before encountering a large tube with radius $r = 2.5$ m which is connected by two bolts. Each bolt can individually sustain a force of $F = 110$ N before failing. What is the maximum distance, d, above the pipe from which the ball can drop so the bolts do not fail?

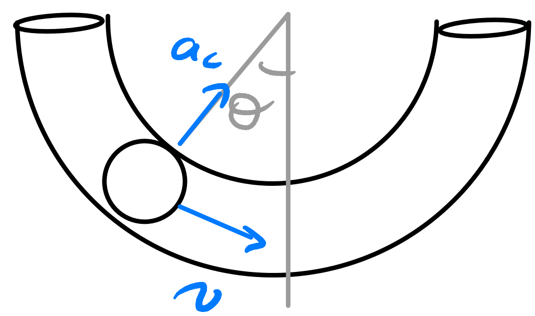


Solution

Goal: Relate d to F!

Let's look at a FBD of the ball in the tube





$$\underline{\sum \vec{F} = m\vec{a}}$$

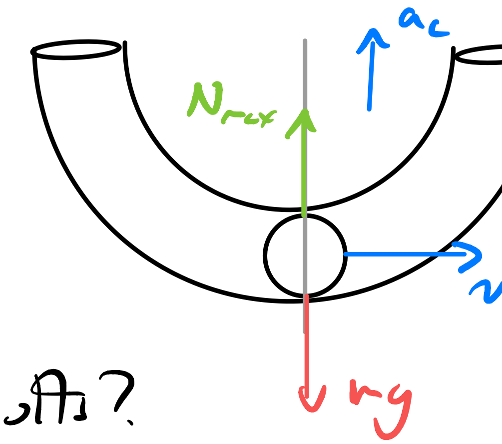
$$y: \quad N - mg\cos\theta = m\frac{v^2}{r}$$

By NIII, Normal force on ball by tube is equal & opposite of force on tube from ball. What is maximum Normal force?

$$N = mg\cos\theta + m\frac{v^2}{r}$$

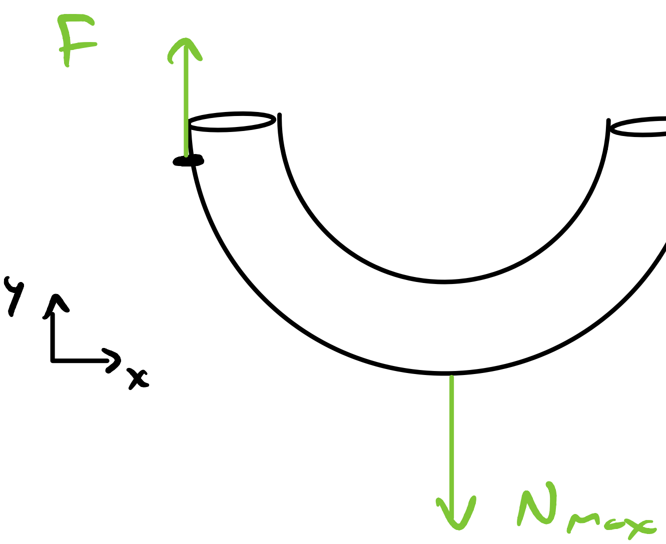
if $\theta = 0°$, N is maximum! $\quad N_{max} = mg + m\frac{v^2}{r}$

$$N_{max} = mg + m\frac{v^2}{r}$$



How to relate this normal force to forces on bolts?

FBD of tube



F = max force on bolt before failure = 110 N

Let's assume tube is massless ($m_{tube} \ll m_{ball}$)

So, $\sum \vec{F} = m\vec{a}$

$$2F - N_{max} = 0 \Rightarrow N_{max} = 2F$$

So, we have the relation

$$2F = mg + m\frac{v^2}{r}$$

Now need v!

We can find v using conservation of energy!

$$E_i = E_f$$

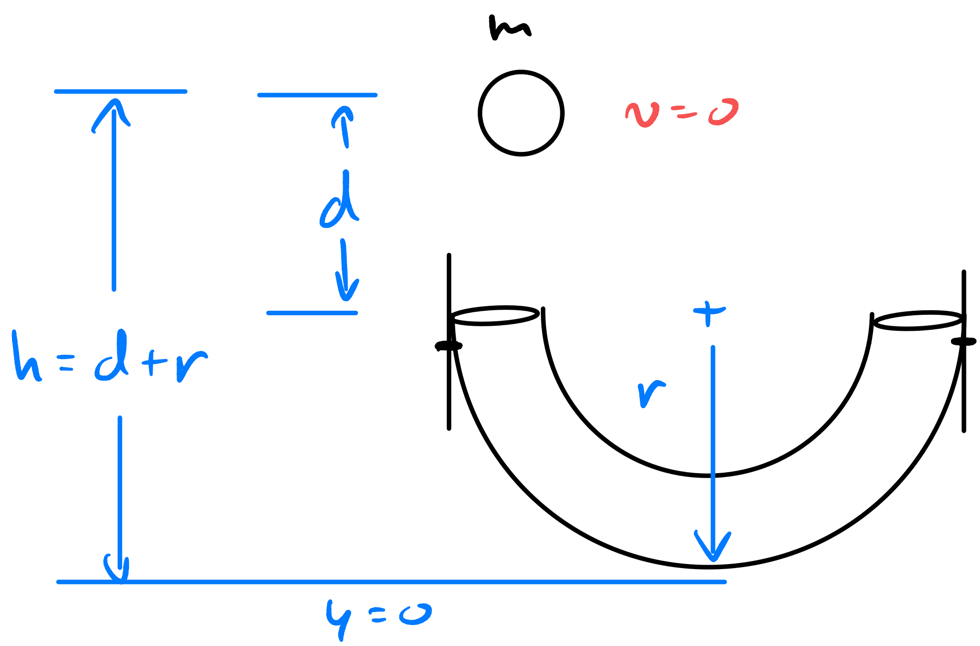
Initial

$E_i = K_i + U_i$

$K_i = 0$

$U_i = mgh$

$\quad = mg(d+r)$

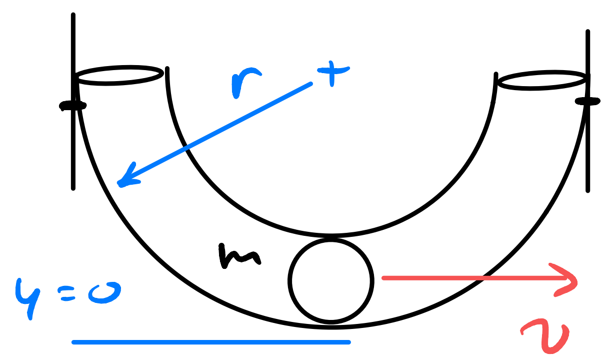


Final

$E_f = K_f + U_f$

$K_f = \frac{1}{2} m v^2$

$U_f = 0$



$\Rightarrow \quad E_i = E_f$

$\Rightarrow \quad mg(d+r) = \frac{1}{2} m v^2$

So, two relations

$$2F = mg + \frac{mv^2}{r} \qquad (1)$$

$$v^2 = 2g(d+r) \qquad (2)$$

So, (2) → (1)

$$2F = mg + \frac{2mg(d+r)}{r}$$

↳ solve for d !

$$\Rightarrow \quad 2Fr - mgr = 2mgd + 2mgr$$

$$\Rightarrow \quad 2mgd = 2Fr - 3mgr$$

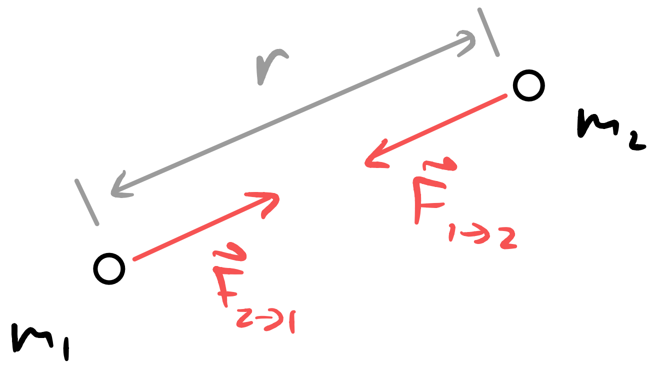
$$\Rightarrow \quad \boxed{d = \frac{Fr}{mg} - \frac{3r}{2}}$$

or, substitute numbers ; $\boxed{d \simeq 21.73 \text{ m}}$

Gravity

Newton's Law of Gravity

$$\vec{F}_{1\to 2} = -G\frac{m_1 m_2}{r^2}\hat{r}$$



from N III

$$\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}$$

$$G = 6.6\times 10^{-11}\ \frac{\text{N m}^2}{\text{kg}^2}$$

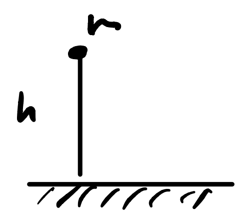
Newton's gravitational constant

Near Earth's surface,

Earth mass

$$\vec{F} = -\frac{GMm}{(R+h)^2}\hat{j}$$

Earth radius



$$\simeq -m\underbrace{\left(\frac{GM}{R^2}\right)}_{\equiv g = 9.8\ \text{m s}^{-2}}\hat{j} + \mathcal{O}\left(\frac{h}{R}\right)$$

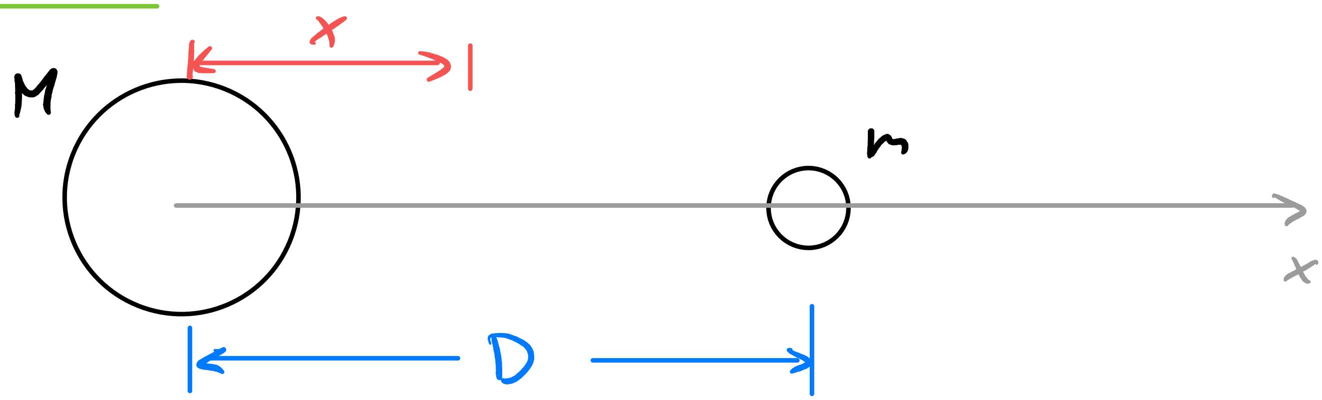
if $h \ll R$, negligible

$$= -mg\hat{j} + \mathcal{O}\left(\frac{h}{R}\right)$$

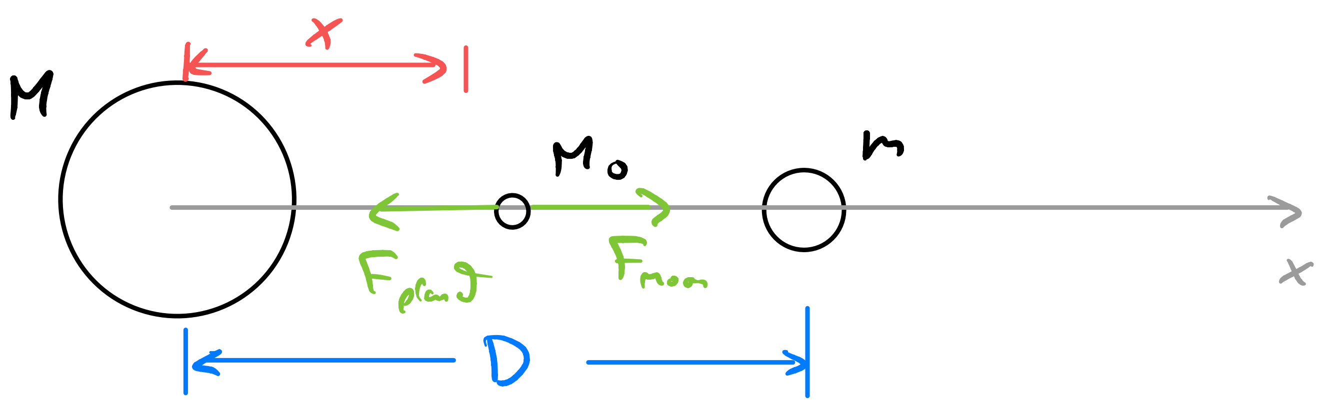
Example

The center of a moon of mass m is a distance D from the center of a planet with mass M. At some distance from the center of the planet, along a line connecting the centers of the planet and moon, the net force will be zero. Where is this location?

Solution



Assume an object with mass m_o at x.



$$\underline{\sum \vec{F} = \vec{0}}$$

$$-F_{planet} + F_{moon} = 0$$

$$F_{planet} = \frac{GMm_0}{x^2}$$

$$F_{moon} = \frac{Gmm_0}{(D-x)^2}$$

$$\Rightarrow \quad -\frac{GMm_0}{x^2} + \frac{Gmm_0}{(D-x)^2} = 0$$

$$\Rightarrow \quad \frac{M}{x^2} = \frac{m}{(D-x)^2}$$

$$\Rightarrow \quad \left(\frac{D-x}{x}\right)^2 = \frac{m}{M}$$

$$\Rightarrow \quad \frac{D}{x} - 1 = \pm\sqrt{\frac{m}{M}}$$

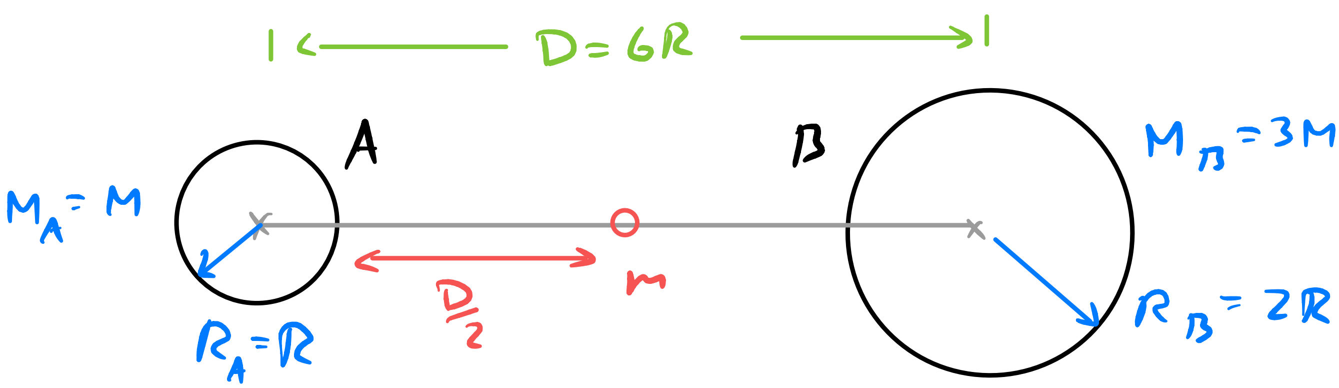
$$\Rightarrow \quad \boxed{x = \frac{D}{1 \pm \sqrt{\frac{m}{M}}}}$$

There are two locations!

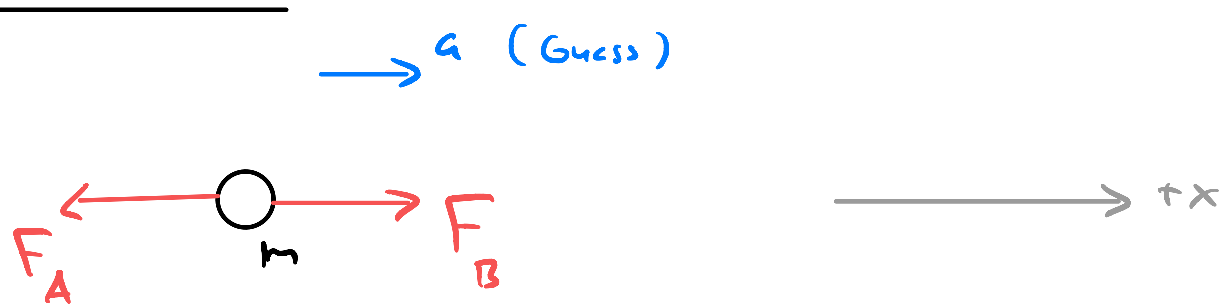
Example

Planet A has a mass M & radius R, while planet B has a mass $3M$ & radius $2R$. They are separated by a distance $6R$. A rock of mass m is released halfway between the planets. Assume the planets do not move. What is the acceleration of the rock?

Solution



FBD of rock



$$\underline{\sum \vec{F} = m\vec{a}}$$

$$F_B - F_A = ma$$

$$F_A = \frac{G M_A m}{r_A^2}$$

Note: Planet radius is NOT important here!

$$F_B = \frac{G M_B m}{r_B^2}$$

$$r_A = \frac{D}{2} = 3R$$

$$r_B = \frac{D}{2} = 3R$$

$$\Rightarrow \quad \frac{G M_B m}{9R^2} - \frac{G M_A m}{9R^2} = ma$$

Now, $M_B = 3M$, $M_A = M$

$$\Rightarrow \quad a = G\left(\frac{M}{3R^2} - \frac{M}{9R^2}\right)$$

$$= \frac{2GM}{9R^2}$$

Since $M > 0$, $R > 0$, acceleration is toward planet B.

$$\boxed{\vec{a} = \frac{2}{9}\frac{GM}{R^2}\,\hat{\imath}}$$