Physics 101 P

General Physics I

Problem Sessions - Week 9

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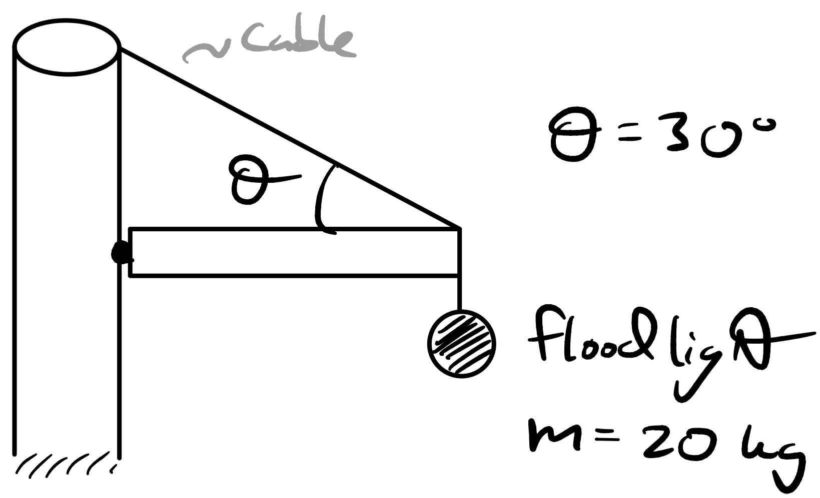
Example

A 20 kg floodlight supported on a beam of negligible mass that is hinged to a pole. A cable at 30° w.r.t. the beam helps support the light.

(a) Find the tension in the cable.

(b) Find the horizontal & vertical force at the hinge.



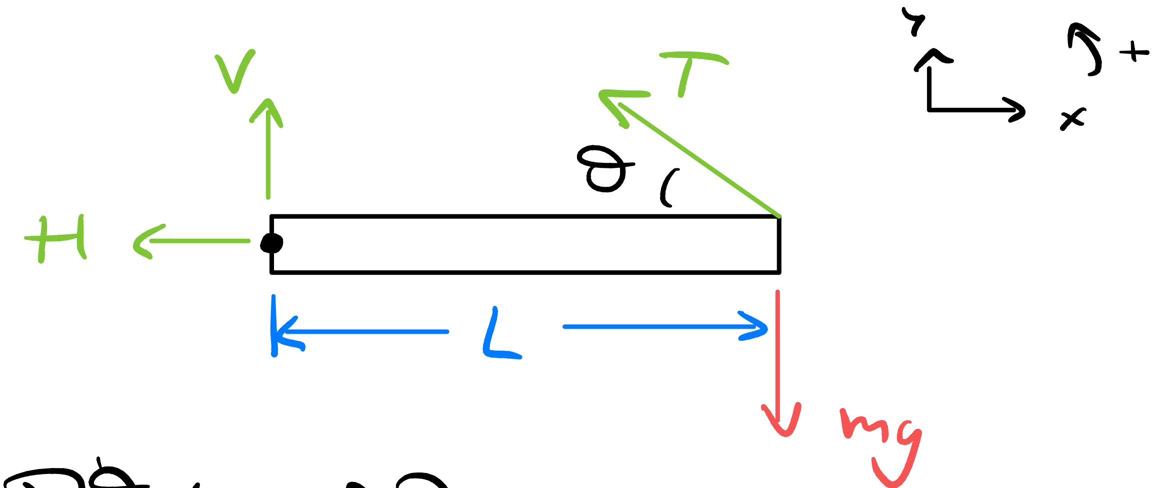
Solution

The beam is in static equilibrium.

$$\sum \vec{F} = \vec{0}$$

$$\sum \tau = 0$$

FBD of beam



Don't know directions of V and H, so guess

(a) Sum torques about hinge

$$+\circlearrowleft \sum \tau = 0 \text{ @ hinge}$$

$$\Rightarrow \quad -Lmg + LT\sin\theta = 0$$

$$\Rightarrow \quad T = \frac{mg}{\sin\theta} \simeq 392 \text{ N}$$ ■

(b) To find H & V, use $\Sigma \vec{F} = \vec{0}$

$x: \quad -H - T\cos\theta = 0$

$y: \quad +V + T\sin\theta - mg = 0$

so, $H = -T\cos\theta$

$\simeq -339 \text{ N}$ ∎

chose wrong direction!

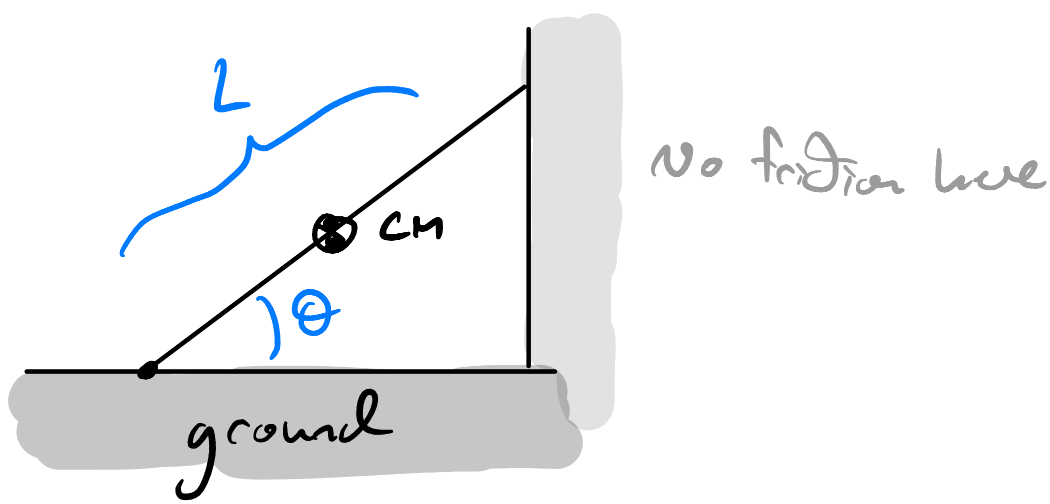
& $V = mg - T\sin\theta$

$= 0$ ⤳ since $T = \dfrac{mg}{\sin\theta}$

$\Rightarrow$ No vertical forces! ∎

Example

A uniform ladder of $L = 10m$ long, weight 50 N, rests on a wall. If the ladder is just on the verge of slipping at $\theta = 50°$ (w.r.t. ground), what is the coefficient of friction between the ladder and the ground?



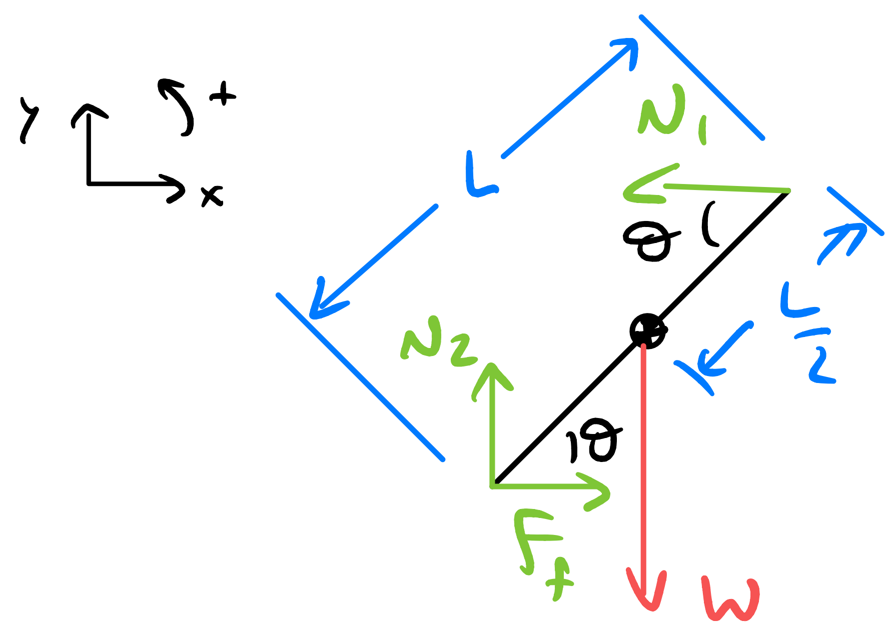
Solution

Ladder is in static equilibrium

$$\sum \vec{F} = \vec{0}$$

$$\sum \tau = 0$$

FBD of ladder



$$\underline{\sum \vec{F} = \vec{0}}$$

x: $F_f - N_1 = 0 \Rightarrow F_f = N_1$ (1)

y: $N_2 - W = 0 \Rightarrow N_2 = W$ (2)

& $F_f = \mu_s N_2$ (3)

So, (2) → (3) → (1)

$\Rightarrow N_1 = \mu_s W$

or $\mu_s = \frac{N_1}{W}$, what is N_1 ?

$+\circlearrowleft \; \underline{\sum \tau = 0}$ @ point where ladder meets ground

$$L N_1 \sin\theta - W \frac{L}{2} \cos\theta = 0$$

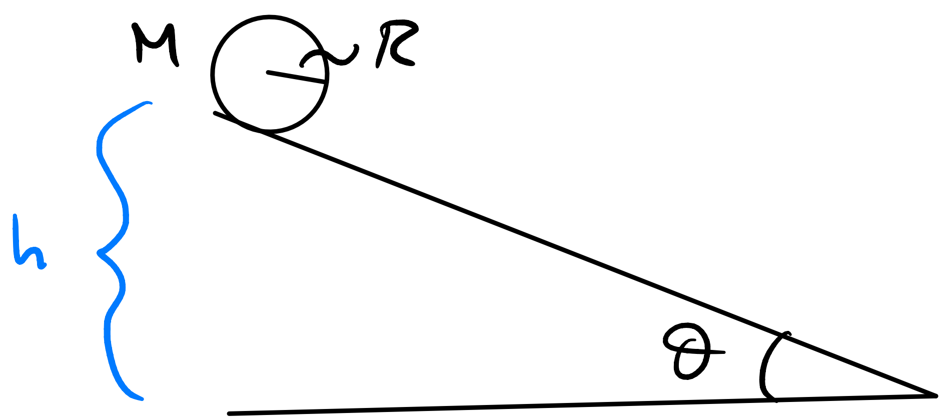
$$\Rightarrow \quad N_1 = \frac{W}{2} \frac{\cos\theta}{\sin\theta} \qquad (4)$$

So,
$$\mu_s = \frac{N_1}{W} = \frac{1}{2} \frac{\cos\theta}{\sin\theta}$$

or,
$$\mu_s = \frac{1}{2} \cot\theta$$
$$= \frac{1}{2} \cot(50^\circ)$$
$$\simeq 0.42 \qquad \blacksquare$$

Example

A sphere of mass M & radius R is released at the top of an inclined plane of height h & angle θ. At the same time, a cylinder of radius R & mass M is also released at the same time. Which reaches the bottom first?



Solution

Let's use conservation of energy.

Initial

$$E_i = U_i + K_i$$
$$= Mgh$$

Final

$$E_f = U_f + K_f$$
$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

The sphere & cylinder roll without slipping

$$v = R\omega \Rightarrow \omega = \frac{v}{R}$$

$$\Rightarrow \quad Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v^2}{R^2}$$

So, $$v^2 = \frac{2gh}{1 + \frac{I}{MR^2}}$$

Now, $I_{sphere} = \frac{2}{5}MR^2$,

$$I_{cylinder} = \frac{1}{2}MR^2$$

So,

$$v_{sphere} = \sqrt{\frac{2gh}{1+\frac{2}{5}}}$$

$$= \sqrt{\frac{10gh}{7}}$$

&

$$v_{cylinder} = \sqrt{\frac{2gh}{1+\frac{1}{2}}}$$

$$= \sqrt{\frac{4gh}{3}}$$

So,

$$\frac{v_{sphere}}{v_{cylinder}} = \frac{\sqrt{10/7}}{\sqrt{4/3}}$$

$$= \sqrt{\frac{30}{28}} > 1$$

$\Rightarrow$ Sphere will reach bottom first! $\blacksquare$

Example

A bug of mass 0.02 kg is at rest on the edge of a solid cylindrical disk ($M = 0.1$ kg, $R = 0.1$ m) rotating in a horizontal plane around the vertical axis through its center. The disk is rotating at 10 rad/s. The bug crawls to the center of the disk.

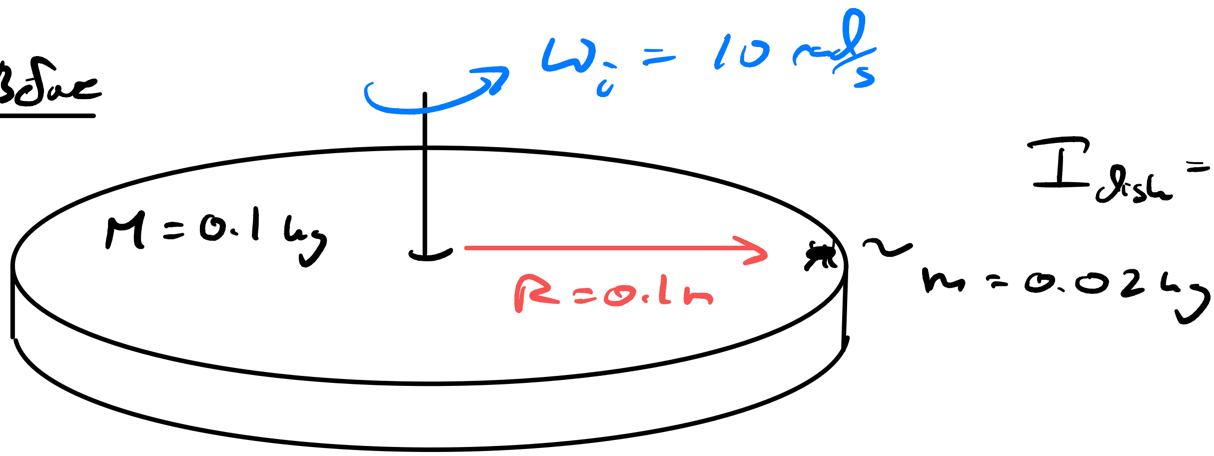
(a) What is the new angular velocity of the disk?

(b) What is the change in kinetic energy of the system?

(c) What is the cause of the increase & decrease of kinetic energy?

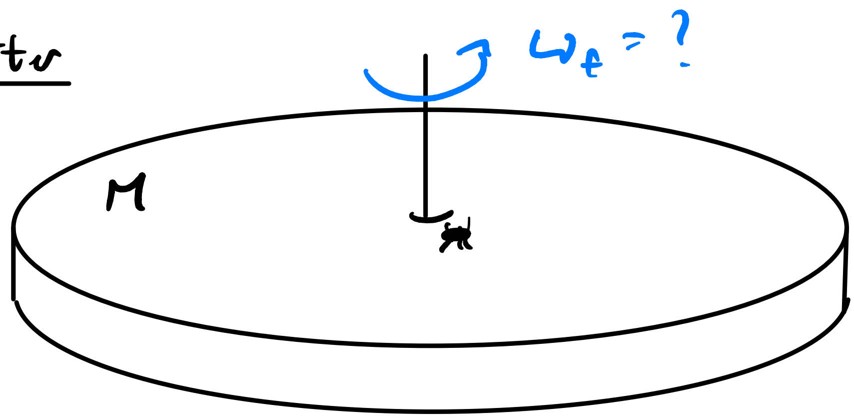
Solution

Before



$$I_{disk} = \frac{1}{2} M R^2$$

After



(a)

Angular momentum is conserved

$$L_i = L_f$$

$$L_i = I \omega_i$$

$$= \left(I_{disk} + m R^2\right) \omega_i$$

$$L_f = I_{disk}\, \omega_f$$

$$\Rightarrow \quad \omega_f = \left(1 + \frac{mR^2}{I_{disk}}\right) \omega_i$$

So,

$$\omega_f = \left(1 + \frac{2m}{M}\right)\omega_i$$

$$= \left(1 + \frac{2(0.02\text{ kg})}{0.1\text{ kg}}\right)(10\text{ rad/s})$$

$$\simeq 14\text{ rad/s} \quad \blacksquare$$

(b)

$$\Delta K = \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I \omega_i^2$$

$$= \frac{1}{2} I_{disk}\left[1 + \frac{mR^2}{I_{disk}}\right]^2 \omega_i^2 - \frac{1}{2}(I_{disk} + mR^2)\omega_i^2$$

$$= \frac{1}{2}\left[\cancel{I_{disk}} + 2mR^2 + \frac{(mR^2)^2}{I_{disk}}\right]\omega_i^2$$

$$- \frac{1}{2}\left[\cancel{I_{disk}} + mR^2\right]\omega_i^2$$

$$= \frac{1}{2} mR^2 \omega_i^2 + \frac{1}{2} \cdot \frac{2m^2R^2}{M}\omega_i^2$$

$$= \frac{1}{2} mR^2 \omega_i^2 \left[1 + \frac{2m}{M}\right] \simeq 0.014\text{ J} \quad \blacksquare$$

(c)

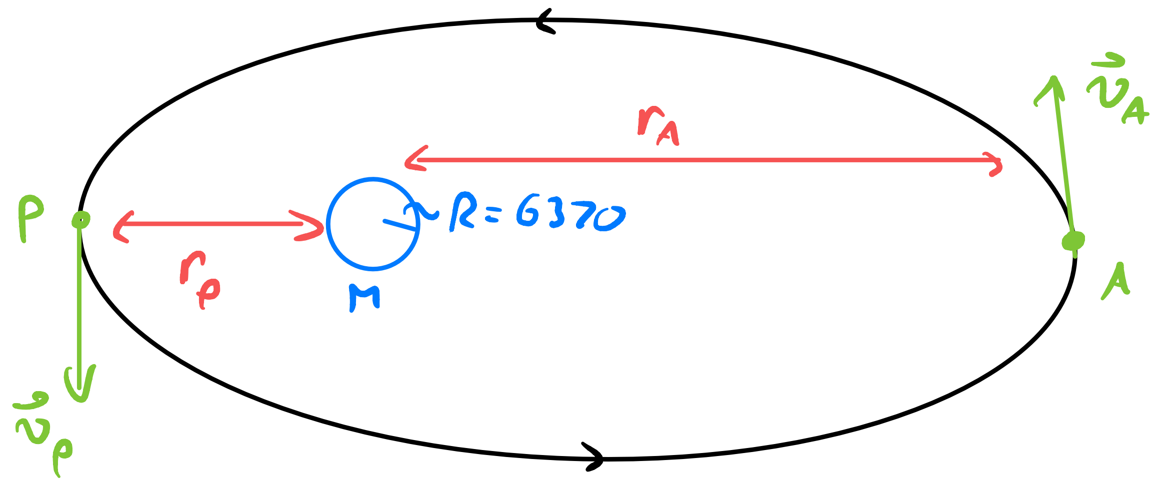
$$W = \Delta K$$

the work is due to non-conservative forces.

Example

An Earth satellite has its apogee at 2500 km above the surface of Earth & perigee at 500 km above the surface of the Earth.

At apogee its speed is 6260 m/s. What is its speed at perigee? Earth's radius is 6370 km.



Solution

Angular momentum is conserved

$$L_A = L_p$$

w/ $L_A = I_A \omega_A$, $L_p = I_p \omega_p$

So, $I_A \omega_A = I_p \omega_p$

Now, $I_A = m r_A^2$, m = mass of satellite

$I_p = m r_p^2$

& $\omega_A = \dfrac{v_A}{r_A}$, $\omega_p = \dfrac{v_p}{r_p}$

So, $r_A^2 \dfrac{v_A}{r_A} = r_p^2 \dfrac{v_p}{r_p}$

$$\Rightarrow \quad v_p = \frac{r_A}{r_p} v_A$$

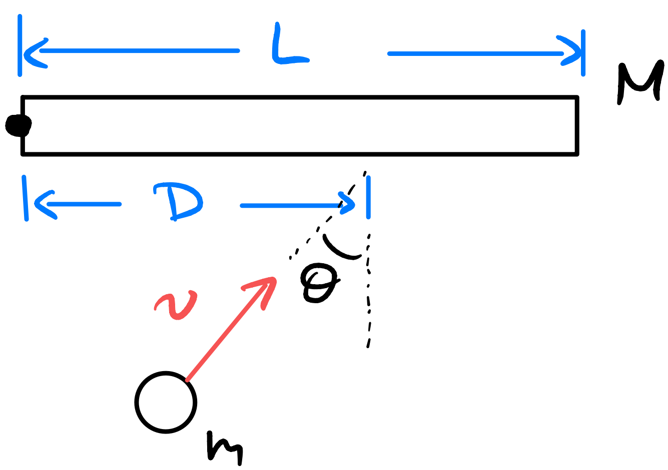
$$= \left(\frac{2500 \text{ km}}{500 \text{ km}} \right) \cdot 6260 \text{ m/s}$$

$$= 5 \cdot 6260 \text{ m/s}$$

$$= 31{,}300 \text{ m/s} \quad \blacksquare$$

Example

A uniform rod of mass M & length L can rotate about a hinge at its left end and is initially at rest. A putty ball of mass m, moving at speed v, strikes the rod at angle θ from the normal & sticks to the rod after the collision. What is the angular speed of the system immediately after the collision?

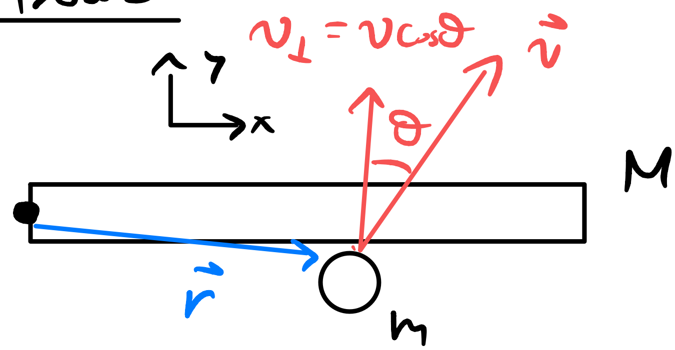


Solution

Angular momentum is conserved

$$L_i = L_f$$

Before

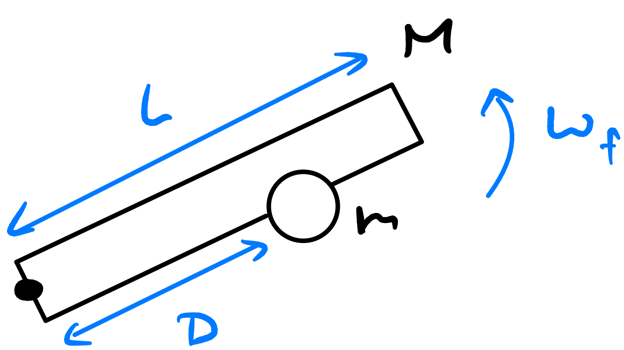


$$\vec{L}_i = \vec{r} \times \vec{p}$$

$$= (D m v \cos\theta)\,\hat{n}$$

$$\Rightarrow \quad L_i = D m v \cos\theta \quad \circlearrowleft$$

After



$$L_f = I\,\omega_f \quad \circlearrowleft$$

$$I = I_{rod} + I_{putty\ ball}$$

Now, from table,

$$I_{rod} = \frac{1}{3} M L^2 \quad \text{about end.}$$

& $$I_{point\ ball} = m D^2$$

$$\Rightarrow \quad I = \frac{1}{3} M L^2 + m D^2$$

So, $$L_f = \left(\frac{1}{3} M L^2 + m D^2 \right) \omega_f$$

Therefore,

$$L_i = L_f$$

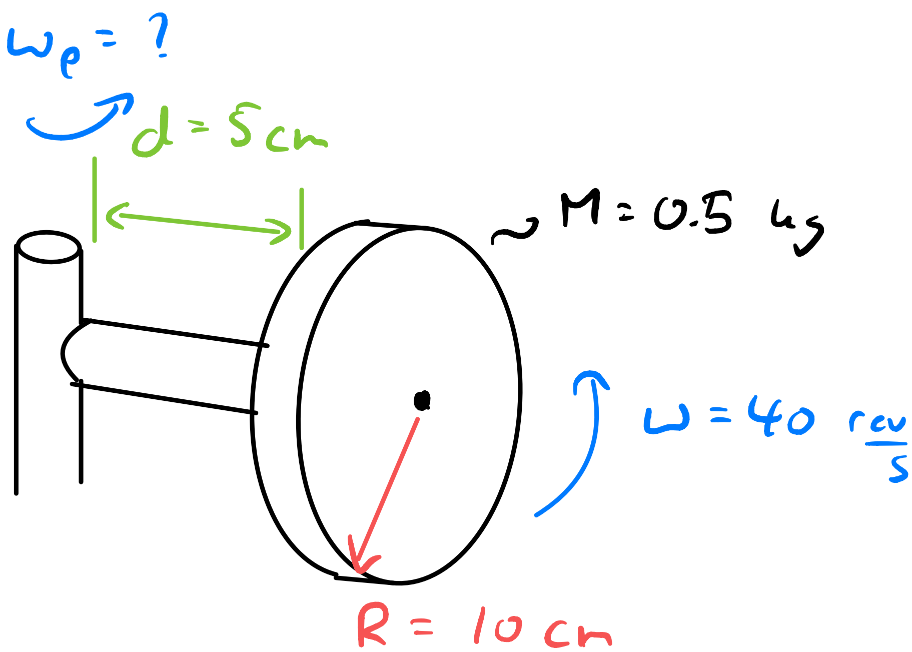
$$\Rightarrow \quad D m v \cos\theta = \left(\frac{1}{3} M L^2 + m D^2 \right) \omega_f$$

or, $$\boxed{\omega_f = \frac{D m v \cos\theta}{\frac{1}{3} M L^2 + m D^2}}$$

Example

A gyroscope has a 0.5 kg disk that spins at 40 rev/s. The center of mass of the disk is 5 cm from a pivot with a radius of the disk of 10 cm. What is the precession angular velocity?

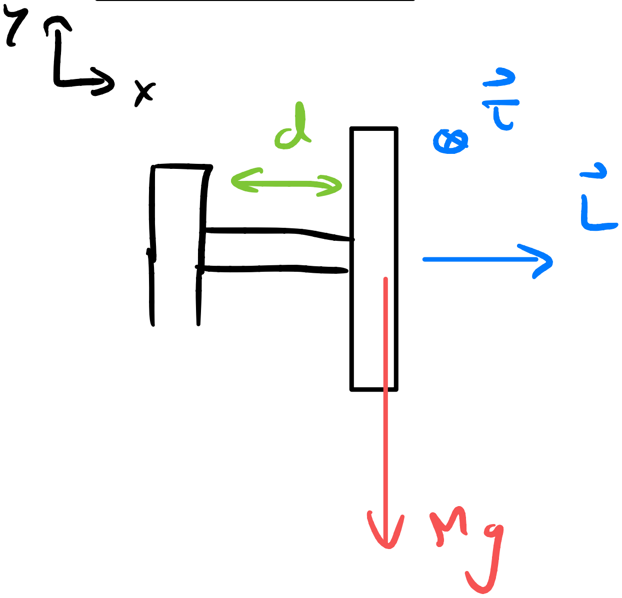
Solution



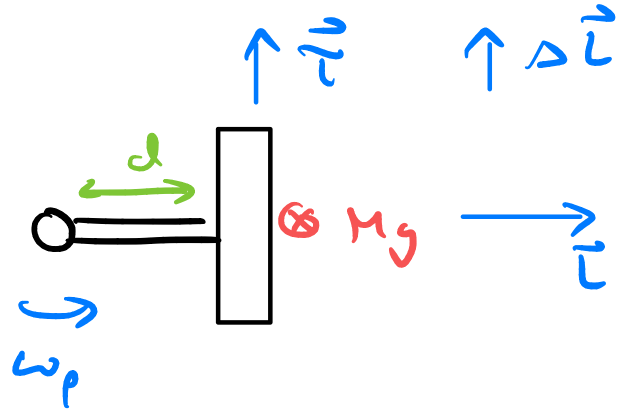
Disk exerts a torque on pivot

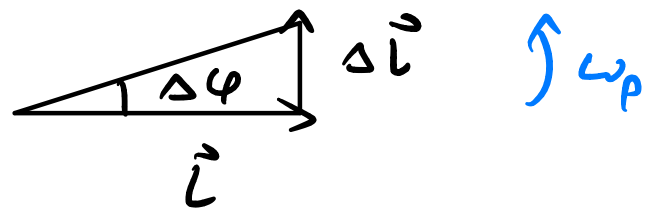
$$L = I_{disk}\,\omega \quad ; \quad I_{disk} = \frac{1}{2} M R^2$$

Side view



Top view





$$\vec{\tau} = \frac{\Delta\vec{L}}{\Delta t}$$

$$\Delta\vec{L} = \vec{L}\,\Delta\varphi$$

$$= \vec{L}\,\frac{\Delta\varphi}{\Delta t} = \vec{L}\,\omega_p$$

$$\omega_p = \frac{\Delta\varphi}{\Delta t}$$

So, $\omega_p = \dfrac{\tau}{L}$

Now, $\tau = dMg$

& $L = \left(\frac{1}{2} M R^2\right)\omega$

So,

$$\omega_p = \frac{\tau}{L} = \frac{dMg}{\left(\frac{1}{2}MR^2\right)\omega}$$

$$= \frac{2dg}{R^2\omega}$$

Need to convert ω,

$$\omega = 40 \frac{\text{rev}}{\text{s}} \cdot \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$

$$= 80\pi \frac{\text{rad}}{\text{s}}$$

So,

$$\omega_p = \frac{2(0.05 \text{ m}) \cdot (9.8 \text{ m/s}^2)}{(0.1 \text{ m})^2 \cdot (80\pi \text{ rad/s})}$$

$$\simeq 0.39 \text{ rad/s} \quad \blacksquare$$

Example

The axis of Earth makes a 23.5° angle with a direction perpendicular to the plane of Earth's orbit. The axis precesses making one complete rotation in 25,780 y.

(a) Calculate the change in angular momentum in half this time.

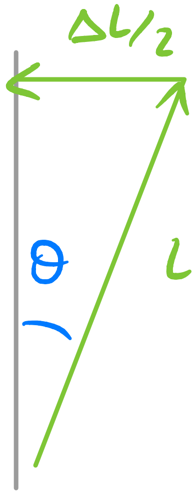
(b) What is the average torque producing this change in angular momentum?

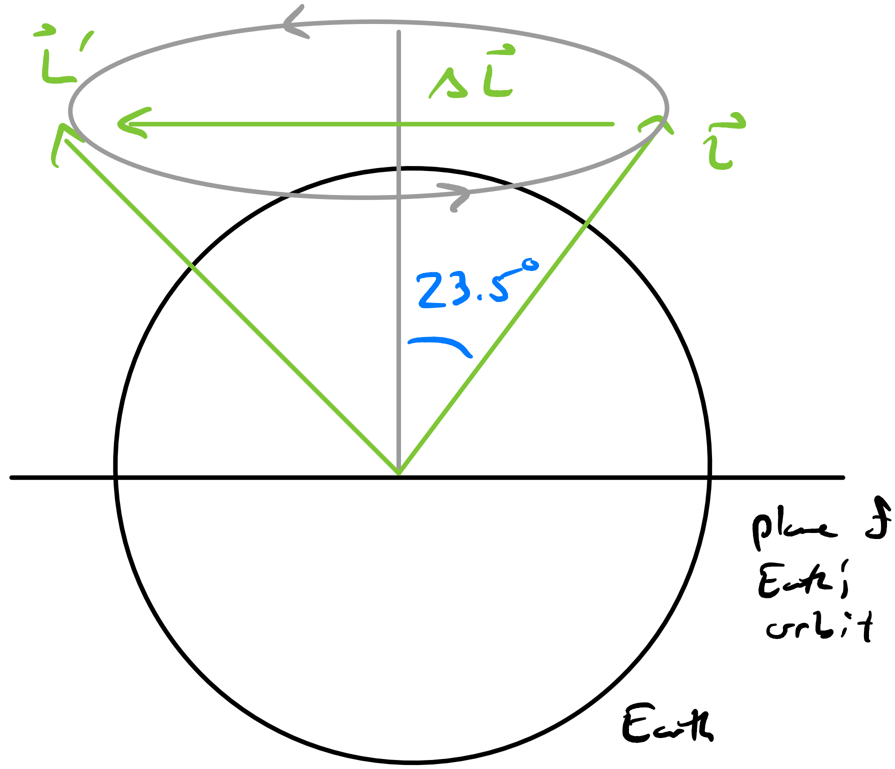
(c) If this torque were created by a pair of forces acting at the most effective point on the equator, what would the magnitude of each force be?

Solution

(a)

From geometry





$$\frac{\Delta L}{2} = L \sin\theta \quad \Rightarrow \quad \Delta L = 2L\sin\theta$$

& $$L = I_{sphere}\,\omega$$

$$I_{sphere} = \frac{2}{5} M R^2$$

$$\omega = \frac{2\pi}{T}$$

$$T = 24\,h \cdot \left(\frac{60\ min}{1\ h}\right) \cdot \left(\frac{60\ s}{1\ min}\right)$$

$$= 86400\ s$$

$$\Rightarrow \quad L = \left(\frac{2}{5} M R^2\right) \cdot \left(\frac{2\pi}{T}\right)$$

$$M = 5.97 \times 10^{24} \text{ kg}$$

$$R = 6.38 \times 10^{6} \text{ m}$$

$$T = 86400 \text{ s}$$

$$\Rightarrow \quad L = 7.07 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}$$

therefore,

$$\Delta L = 2L \sin\theta$$

$$= 2\left(7.07 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s}\right) \sin(23.5°)$$

$$\simeq 5.64 \times 10^{33} \text{ kg} \cdot \text{m}^2/\text{s} \quad \blacksquare$$

(b) $\tau = \dfrac{\Delta L}{\Delta t}$

$$\Delta t = \frac{1}{2}(25{,}780 \text{ y}) \cdot \left(\frac{365.25 \text{ d}}{\text{y}}\right)\left(\frac{24 \text{ h}}{\text{d}}\right)\left(\frac{60 \text{ min}}{\text{h}}\right)\left(\frac{60 \text{ s}}{\text{min}}\right)$$

$$= 4.07 \times 10^{11} \text{ s}$$

So,

$$\tau = \frac{\Delta L}{\Delta t}$$

$$\simeq 1.39 \times 10^{22} \ N \cdot m$$

(c)

$$\tau = 2RF$$

$$\Rightarrow F = \frac{\tau}{2R}$$

$$\simeq 1.09 \times 10^{15} \ N$$