Physics 101 P

General Physics I

Problem Sessions - Week 10

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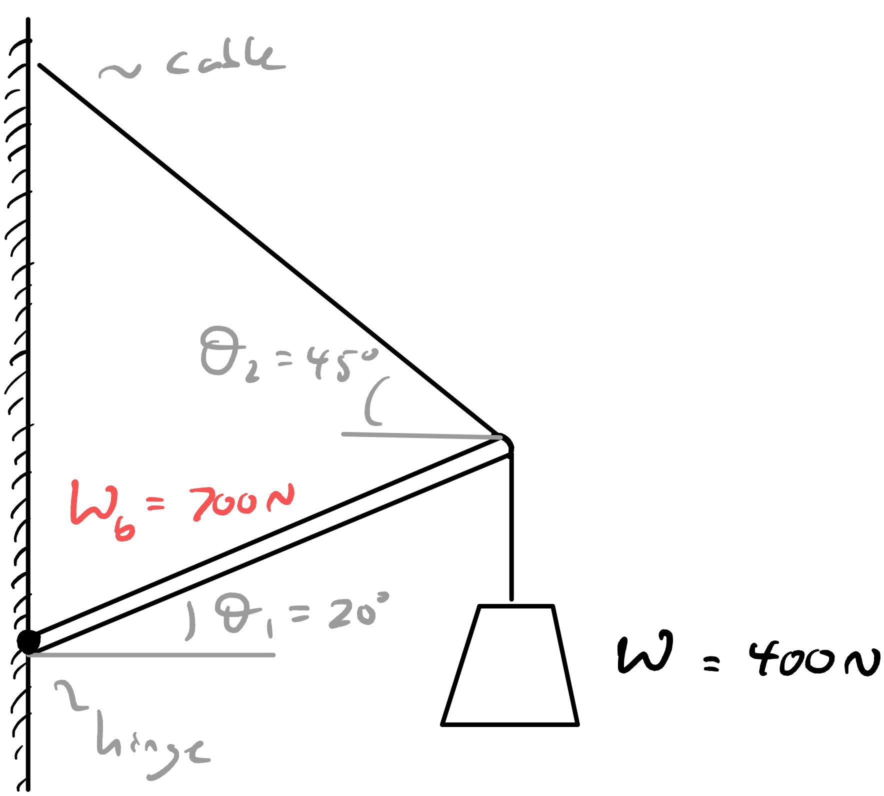
Example

The uniform boom weighs 700 N, & the object hanging from its right end weighs 400 N. The boom is supported by a light cable & by a hinge at the wall. Calculate the tension in the cable & the force on the hinge on the boom.



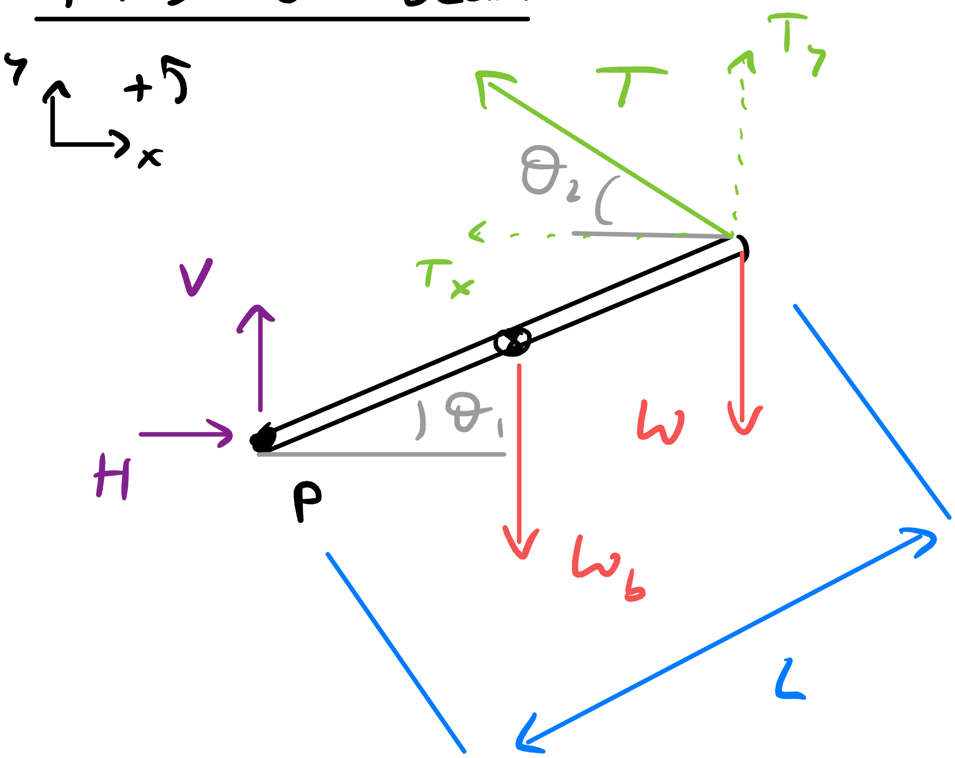
Solution

Static equilibrium

$$\sum \vec{F} = \vec{0}$$

$$\sum \vec{\tau} = 0$$

FBD of beam



$T_x = T\cos\theta_2$

$T_y = T\sin\theta_1$

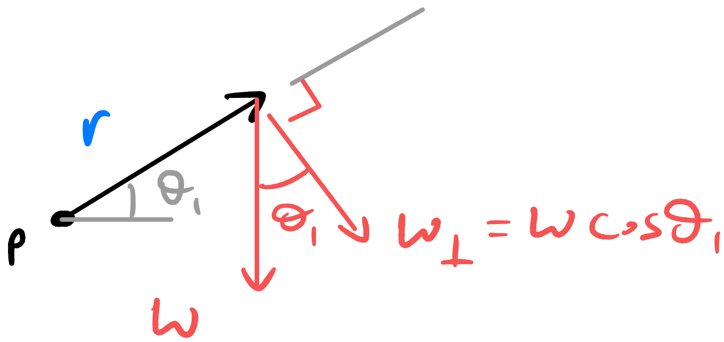
$\sum \vec{F} = \vec{0}$

$x: \quad H - T\cos\theta_2 = 0$

$y: \quad V + T\sin\theta_2 - W - W_b = 0$

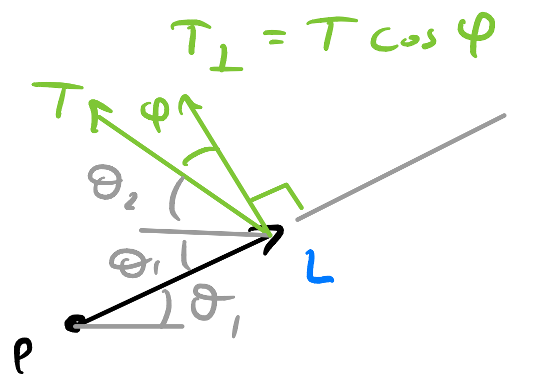
$$\sum \tau_p = 0$$

Torque from weight, θ_1



$$\Rightarrow \tau = rW\cos\theta_1$$

Torque from Tension



$$\varphi = 90^\circ - \theta_1 - \theta_2$$
$$= 25^\circ$$

$$\Rightarrow \tau = LT\cos\varphi$$

$$\Rightarrow \quad -\frac{L}{2}W_b\cos\theta_1 - LW\cos\theta_1 + LT\cos\varphi = 0$$

$$\Rightarrow \quad T\cos\varphi - \left(\frac{W_b}{2} + W\right)\cos\theta_1 = 0$$

So, 3 equations, 3 unknowns (H, V, T)

$$H - T\cos\theta_2 = 0 \qquad (1)$$

$$V + T\sin\theta_2 - W - W_b = 0 \qquad (2)$$

$$T\cos\varphi - \left(\frac{W_b}{2} + W\right)\cos\theta_1 = 0 \qquad (3)$$

From (3)

$$T = \left(W + \frac{W_b}{2}\right)\frac{\cos\theta_1}{\cos\varphi}$$

$$\simeq 777.6 \text{ N} \quad \blacksquare$$

From (1)

$$H = T\cos\theta_2$$

$$\simeq 549.8 \text{ N} \quad \blacksquare$$

From (2

$$V = W + W_b - T\sin\theta_2$$

$$\simeq 550.1 \text{ N} \quad \blacksquare$$

Example

The cable in the previous example is made of steel and has an ultimate tensile strength of 500 MPa. If the cable has a diameter of 2 cm, does the cable break?

Solution

We want to see if the tensile stress, σ_T, is greater than the ultimate tensile stress, $\sigma|_{UT}$.

Recall

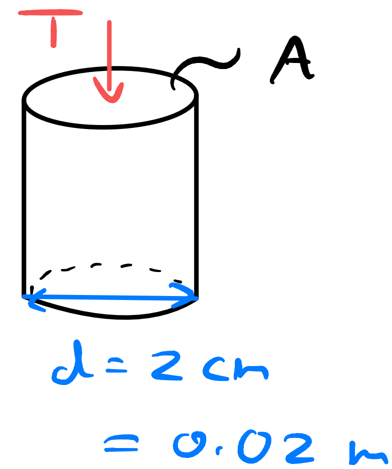
$$\text{tensile stress} = \sigma_T = \frac{F}{A}$$

For this case, we know

$$T = 777.6\ N$$

& we have a circular cross-section

$$A = \pi r^2 = \frac{\pi d^2}{4}$$



So,

$$\sigma_T = \frac{T}{\left(\frac{\pi d^2}{4}\right)}$$

$$= \frac{777.6\ N}{\frac{\pi}{4}(0.02\,m^2)}$$

$$\simeq 2.48 \times 10^6\ \frac{N}{m^2}$$

$$= 2.48 \times 10^6\ Pa$$

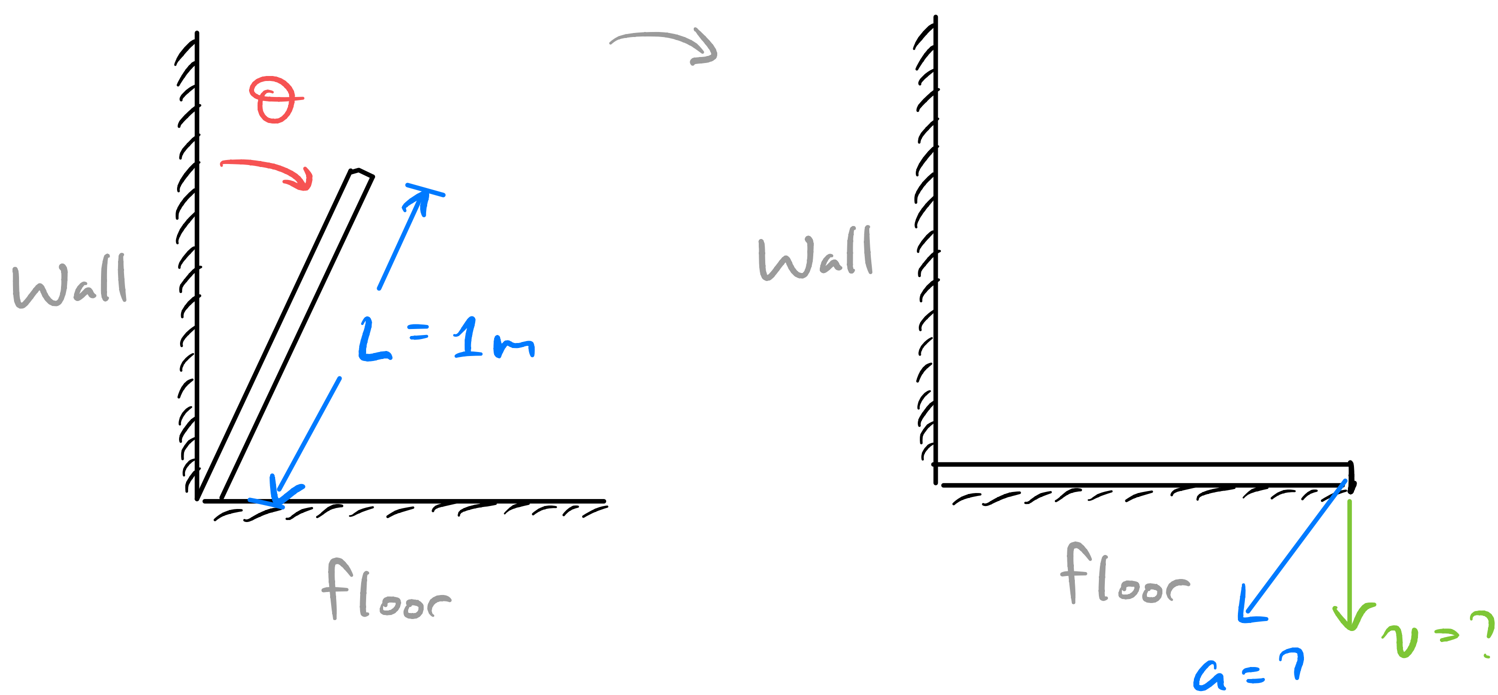
Now, $\sigma_{UT} = 500\ MPa = 500 \times 10^6\ Pa$

$\rightarrow \quad \sigma_T < \sigma_{UT} \quad \Rightarrow$ cable does not break!

Example

A meter stick (assume a uniformly thin rod) is initially at rest vertically against a wall as shown. The rod then starts to fall over. What is the magnitude of the speed and acceleration at the end of the meter stick as it hits the ground?

Given $I_{cm} = \frac{1}{12} M L^2$



Solution

Since the meter stick is up against the wall, the end of the rod is effectively "hinged", i.e., it cannot slide, just rotate.

From rotational kinematics

$$v = L\omega$$

& $$\vec{a} = \vec{a}_t + \vec{a}_c$$

$$\Rightarrow a = \sqrt{a_t^2 + a_c^2}$$

w/ $a_t = L\alpha$ tangential acceleration

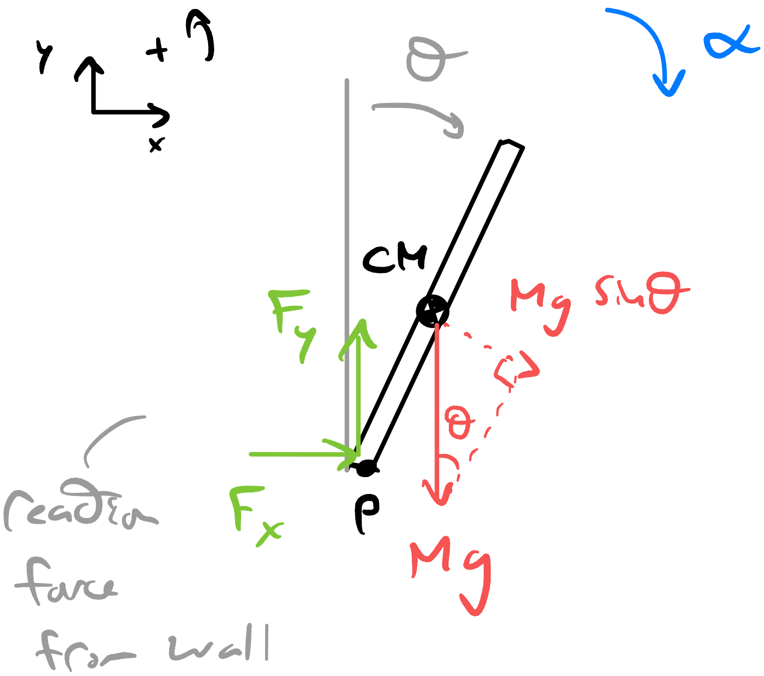
$a_c = L\omega^2$ centripetal acceleration

So, $$a = L\sqrt{\alpha^2 + \omega^4}$$

$\therefore$ we need to calculate ω & α about the pivot point.

First, let's calculate α.

FBD of stick



$$\sum \tau_p = I_p \alpha$$

$$-Mg\left(\frac{L}{2}\right)\sin\theta = -I_p\alpha$$

$$\Rightarrow \alpha = \frac{MgL}{2I_p}\sin\theta$$

We know $I_{cm} = \frac{1}{12}ML^2$

To find I_p, we parallel axis theorem

$$I_p = I_{cm} + MD^2$$

here, $D = \frac{L}{2}$

$$\Rightarrow \quad I_p = \frac{1}{12}ML^2 + \frac{ML^2}{4} = \frac{1}{3}ML^2$$

So,

$$\alpha = \frac{MgL}{2I_p} \sin\theta$$

$$= \frac{3}{2}\frac{g}{L} \sin\theta$$

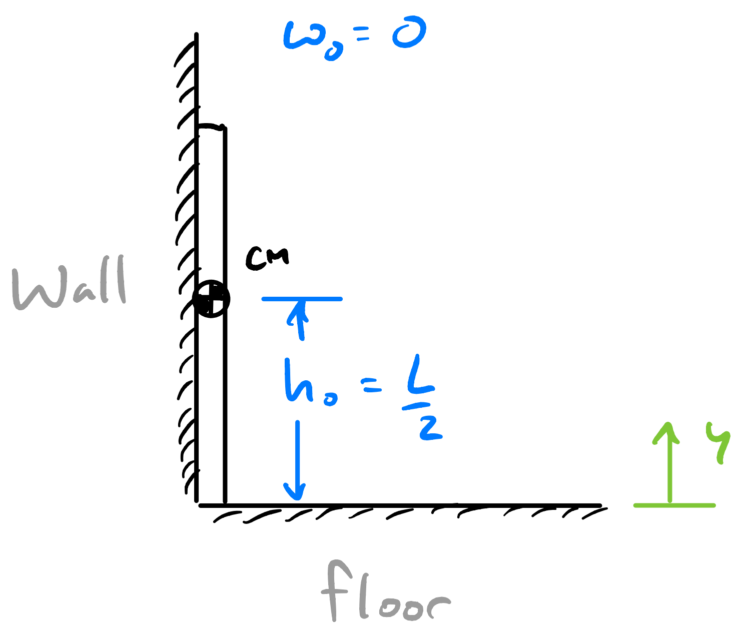
When the stick hits the ground,

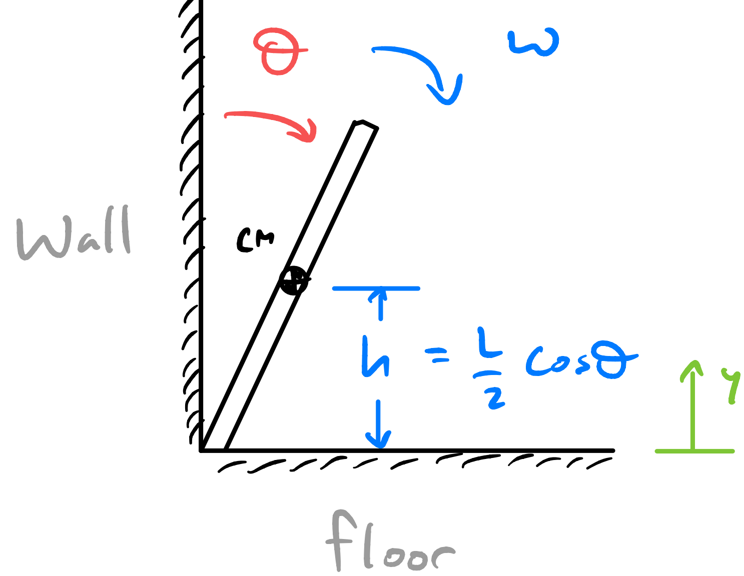
$$\theta = 90° = \frac{\pi}{2} \Rightarrow \sin\theta = 1$$

$$\Rightarrow \alpha\Big|_{ground} = \frac{3g}{2L}$$

Next, we need to find ω.

Let's use conservation of energy





$$E_i = U_i + K_i$$

$$= Mg\frac{L}{2}$$

$$E_f = U_f + K_f$$

$$= Mg\frac{L}{2}\cos\theta + \frac{1}{2}I_p\omega^2$$

$$\Rightarrow \omega^2 = \frac{MgL}{I_p}(1-\cos\theta)$$

$$= \frac{3g}{L}(1-\cos\theta)$$

$$\Rightarrow \omega = \sqrt{\frac{3g}{L}(1-\cos\theta)}$$

Now, at ground, $\theta = 90° = \frac{\pi}{2} \Rightarrow \cos\theta = 0$

$$\Rightarrow \omega\Big|_{ground} = \sqrt{\frac{3g}{L}}$$

therefore,

$$v\big|_{ground} = L\omega\big|_{ground}$$

$$= \sqrt{3gL}$$

Since $L = 1\,m$,

$$\Rightarrow v\big|_{ground} = 5.42 \; m/s \quad \blacksquare$$

and,

$$a\big|_{ground} = L\sqrt{\alpha^2 + \omega^4}$$

$$= L\sqrt{\left(\frac{3g}{2L}\right)^2 + \left(\frac{3g}{L}\right)^2}$$

$$= 3g\sqrt{1 + \frac{1}{4}}$$

$$= \frac{3\sqrt{5}}{2}\,g$$

$$\approx 32.9 \; m/s^2 \quad \blacksquare$$

Alternative way to get ω

recall

$$\alpha = \frac{d\omega}{dt}$$

but, don't have time-dependence. so, use

$$\frac{d\omega}{dt} = \frac{d\theta}{dt}\frac{d\omega}{d\theta}$$

$$= \omega \frac{d\omega}{d\theta}$$

$$\Rightarrow \quad \alpha = \omega \frac{d\omega}{d\theta}$$

Now, integrate

$$\int_0^{\theta} \alpha \, d\theta = \int_0^{\omega} \omega \, d\omega$$

$$= \frac{1}{2}\omega^2$$

Now,
$$\alpha = \frac{3g}{2L} \sin\theta$$

$$\Rightarrow \int_0^{\theta} \frac{3g}{2L} \sin\theta \, d\theta = \frac{3g}{2L} \left(-\cos\theta \right|_0^{\theta}$$

$$= \frac{3g}{2L} (1 - \cos\theta)$$

$$\Rightarrow \frac{1}{2}\omega^2 = \frac{3g}{2L} (1 - \cos\theta)$$

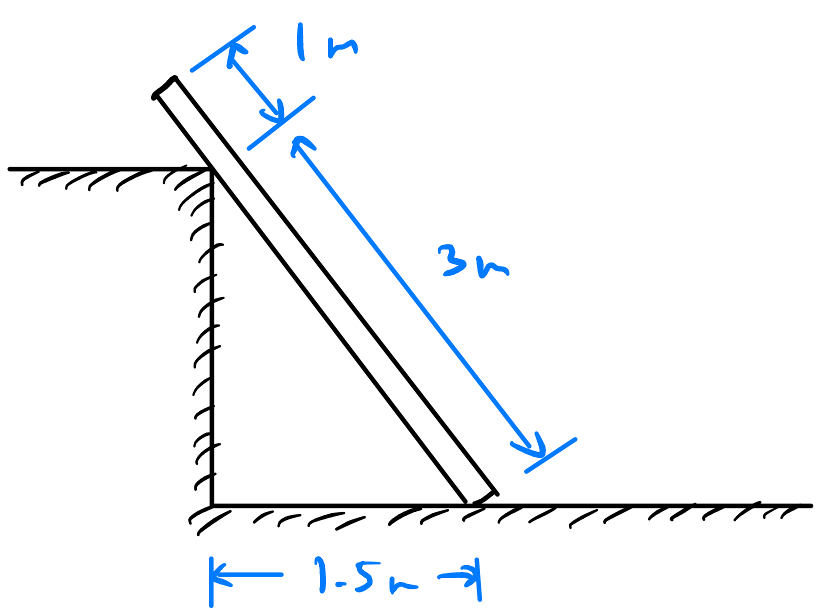
$$\Rightarrow \omega^2 = \frac{3g}{L} (1 - \cos\theta)$$

Example

A uniform 4m plank weighing 200 N rests against the corner of a wall. There is no friction at the point where the plank meets the corner.

(a) Find the forces that the corner and the floor exert on the plank.

(b) What is the minimum coefficient of static friction between the floor & the plank to prevent the plank from slipping?



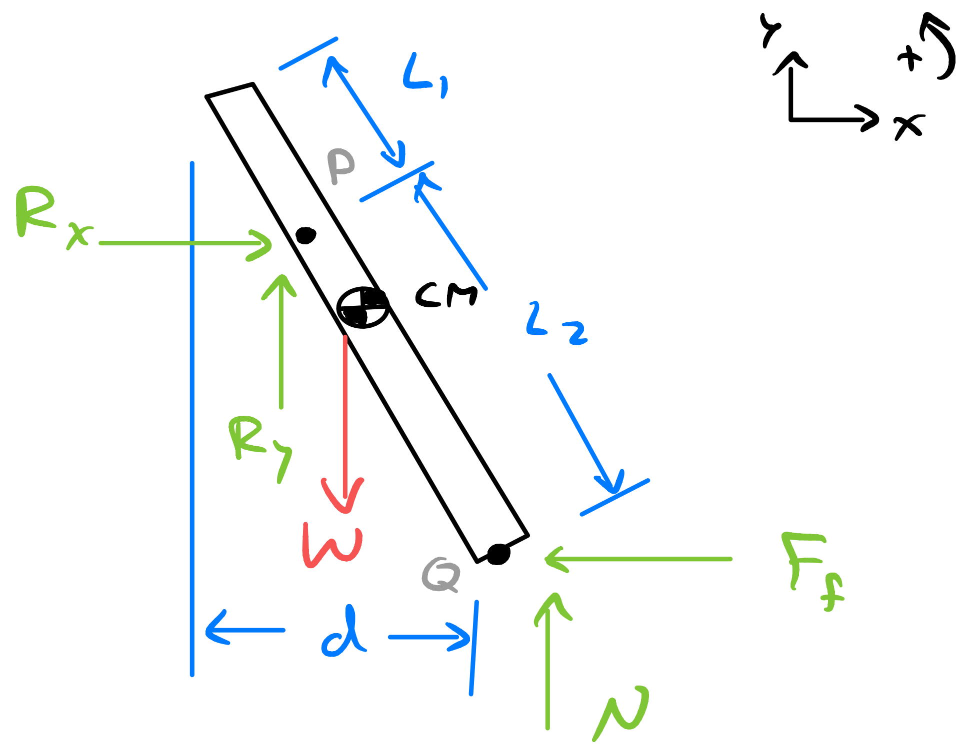
Solution

Static equilibrium

$$\sum \vec{F} = \vec{0}$$

$$\sum \tau = 0$$

FBD of plank



$\underline{\sum \vec{F} = \vec{0}}$

x : $\quad R_x - F_f = 0 \qquad (1)$

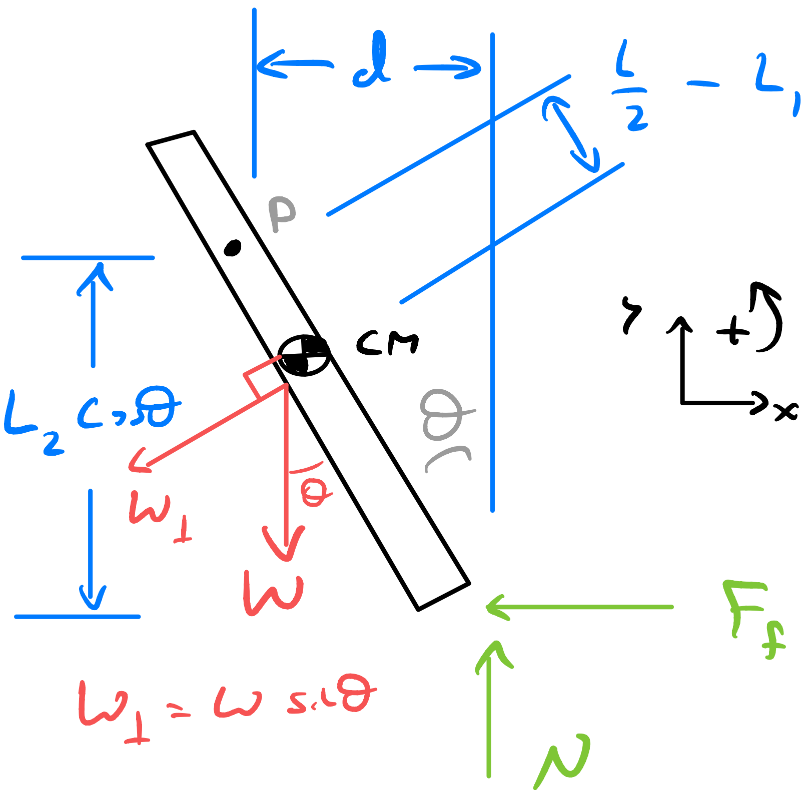
y : $\quad R_y - W + N = 0 \qquad (2)$

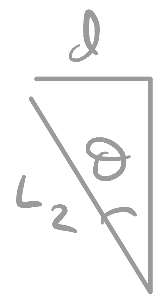
Sum torques at point P and point Q.

Note: $L = L_1 + L_2$; $L_1 = \frac{1}{4} L$

$L_2 = \frac{3}{4} L$

@ point P





$$\sin\theta = \frac{d}{L_2}$$

$$\theta = \sin^{-1}\left(\frac{d}{L_2}\right) \simeq 30^\circ$$

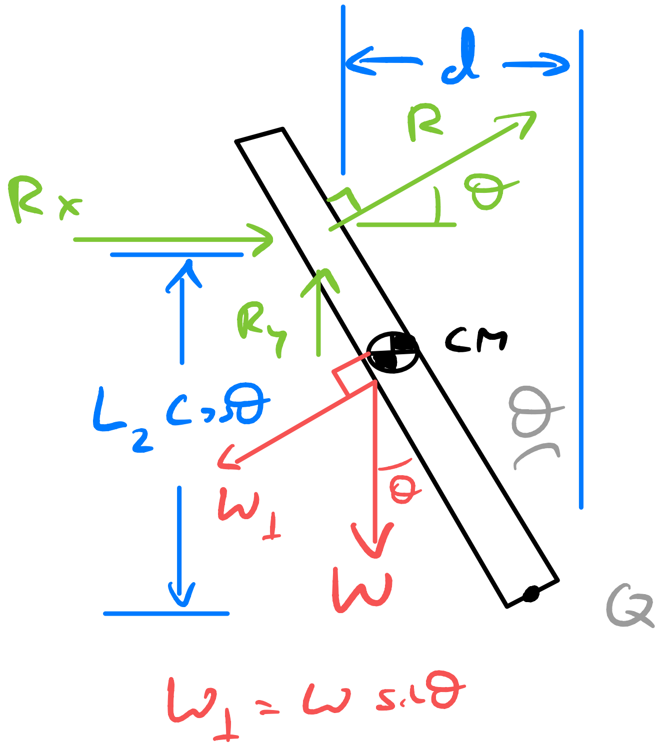
$\sum \tau_p = 0$

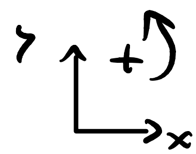
$$-W\left(\frac{L}{2} - L_1\right)\sin\theta + Nd - F_f L_2 \cos\theta = 0$$

Note: $\frac{L}{2} - L_1 = \frac{1}{2}L - \frac{1}{4}L = \frac{1}{4}L$

$$\Rightarrow -W\frac{L}{4}\sin\theta + Nd - F_f L_2 \cos\theta = 0 \qquad (3)$$

@ point Q





Either work with R or R_x, R_y.

Easier here to work with R, & use

$R_x = R\cos\theta$

$R_y = R\sin\theta$

$\underline{\sum \tau_Q = 0}$

$$W \frac{L}{2} \sin\theta - R L_2 = 0 \qquad (4)$$

So, 4 eqns, 4 unknowns (R_x, R_y, N, F_f)

$$R_x - F_f = 0 \qquad (1)$$

$$R_y - W + N = 0 \qquad (2)$$

$$-\frac{WL}{4}\sin\theta + Nd - F_f L_2 \cos\theta = 0 \qquad (3)$$

$$W\frac{L}{2}\sin\theta - R L_2 = 0 \qquad (4)$$

$$\theta = 30° \Rightarrow \sin\theta = \frac{1}{2}$$
$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad R_x - F_f = 0$$

$$R_y - W + N = 0$$

$$-\frac{WL}{4} + Nd - \frac{\sqrt{3}}{2} F_f L_2 = 0$$

$$\frac{WL}{4} - R L_2 = 0$$

From (4), $R = \frac{W}{4}\frac{L}{L_2}$

Now, $L_2 = \frac{3L}{4}$

$$\Rightarrow \quad R = \frac{W}{4} \frac{L}{\frac{3}{4}L}$$

$$= \frac{W}{3}$$

$$\simeq 66.7 \text{ N} \blacksquare$$

Now,

$$R_x = R\cos\theta = \frac{\sqrt{3}}{2} R \simeq 57.7 \text{ N} \blacksquare$$

$$R_y = R\sin\theta = \frac{1}{2} R \simeq 33.3 \text{ N} \blacksquare$$

Now, from (1)

$$F_f = R_x = 57.7 \text{ N} \blacksquare$$

& from (2)

$$N = W - R_y$$

$$= 166.7 \text{ N} \blacksquare$$

(b)

For μ_s, note

$$F_f = \mu_s N$$

So,

$$\mu_s = \frac{F_f}{N} \simeq \frac{57.7\,N}{166.7\,N} = 0.35 \quad \blacksquare$$