Physics 101 P

General Physics I

Problem Sessions – Week 11

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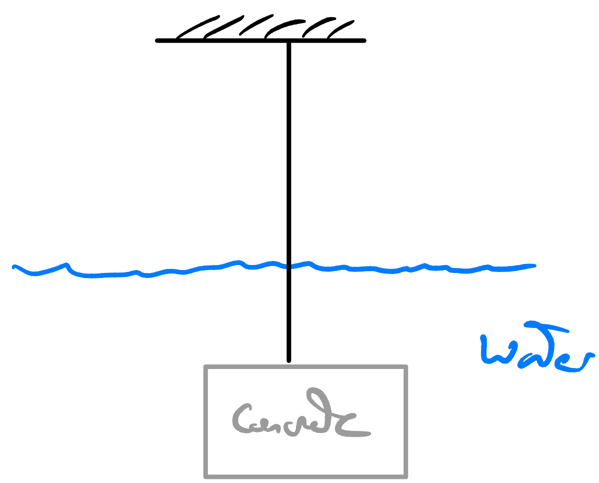
Example

A 500 kg solid block of concrete is submerged under water and held by an ideal cable, as shown. The density of concrete is 2300 kg/m³, & the density of water is 1000 kg/m³. What is the buoyant force on the block? What is the tension of the cable? Note that the tension measures the apparent weight.

Solution



The buoyant force is given by

$$B = \rho_{H_2O} V g$$

where V is volume of displaced water

Now, the volume of concrete is the same volume of water displaced as the concrete is fully submerged

$\Rightarrow$ Archimedes' principle

$$\Rightarrow \quad m = \rho_{concrete} V \Rightarrow V = \frac{m}{\rho_{concrete}} = 0.217 \ m^3$$

So,

$$B = \rho_{H_2O} V g$$

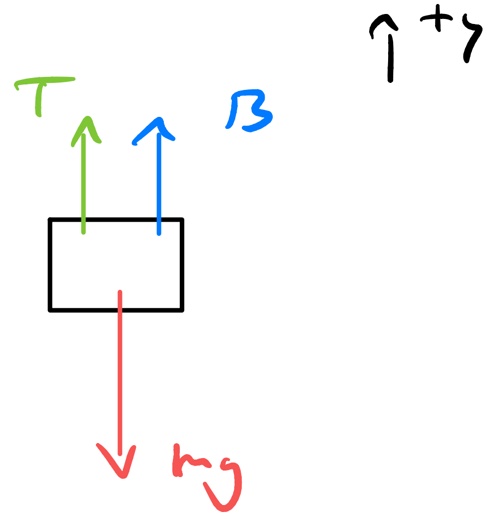
$$= \frac{\rho_{H_2O}}{\rho_{concrete}} mg$$

$$\simeq \frac{mg}{2.3}$$

$$\simeq 0.43 \, mg = 2,170 \ N$$

43% of weight of concrete

The tension on cable can be found by $\sum \vec{F} = 0$



$$\underline{\sum \vec{F} = 0}$$

$$y: \; T + B - mg = 0$$

$$\Rightarrow \; T = mg - B$$

$$= mg\left(1 - \frac{\rho_{H_2O}}{\rho_{concrete}}\right)$$

$$= 0.57\,mg \quad \leftarrow \text{57\% of weight of concrete}$$

$$\simeq 2800 \text{ N} \quad \blacksquare$$

What if not H_2O, but air

$$\rho_{air} = 1.3 \; \frac{kg}{m^3}$$

$$\Rightarrow \; T = \left(1 - \frac{\rho_{air}}{\rho_{concrete}}\right) mg$$

$$\simeq 0.9994 \; mg$$

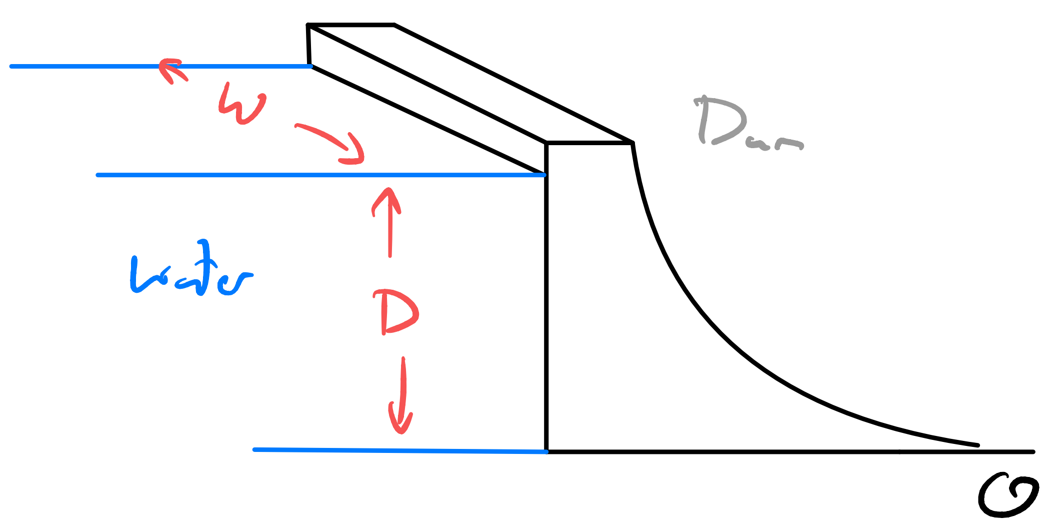
negligible effect

Example

Water stands at a depth D behind the vertical upstream face of a dam. Let W be the width of the dam.

(a) Find the resultant horizontal force exerted on the dam by the gauge pressure of the water.

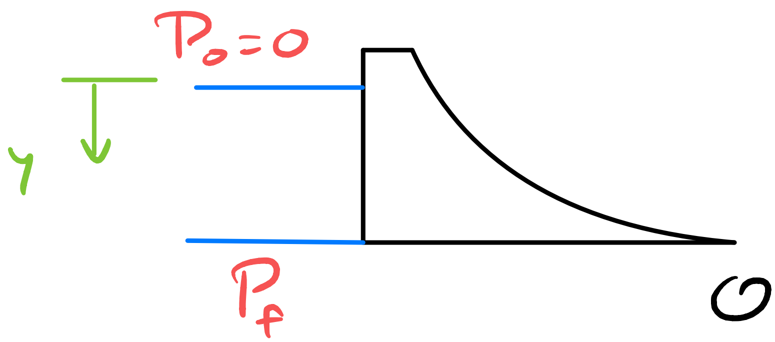
(b) Find the net torque due to the gauge pressure of the water exerted about a line through O parallel to the width of the dam.



Solution

Gauge pressure = pressure relative to atmospheric

(a)

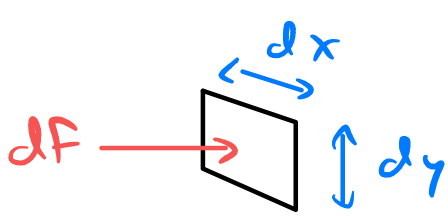


Now, $P - P_0 = \rho g (y - y_0)$

@ $y_0 = 0$, $P_0 = 0$

$\Rightarrow P = \rho g y$

Look at infinitesimal area on dam



$dF = P\,dx\,dy$

So,

$$F = \int P\,dx\,dy$$

$$= \rho g \int_0^W dx \int_0^D y\,dy$$

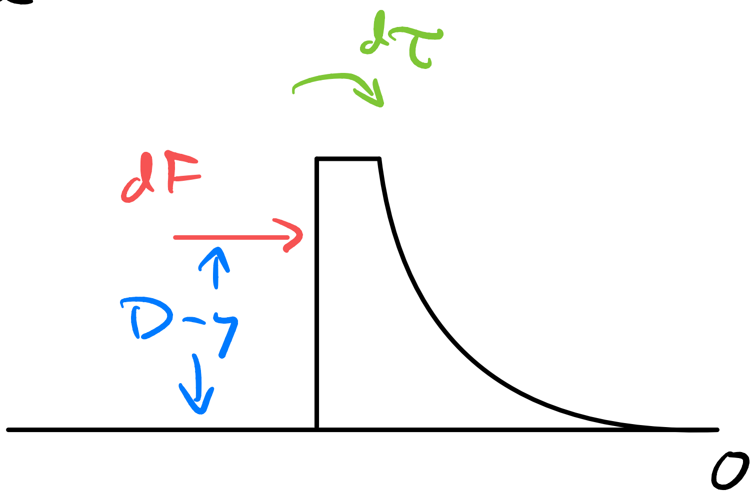
$$= \rho g W \left.\frac{y^2}{2}\right|_0^D$$

$$= \frac{1}{2}\rho g W D^2 \quad \blacksquare$$

As we go deeper, larger force

$\Rightarrow$ Dam needs to be thicker

(b) torque



$$d\tau = (D-y)\,dF$$

$$\Rightarrow \quad \tau = \int (D-y)\,P\,dx\,dy$$

$$= \rho g \int_0^W dx \int_0^D (D-y)\,y\,dy$$

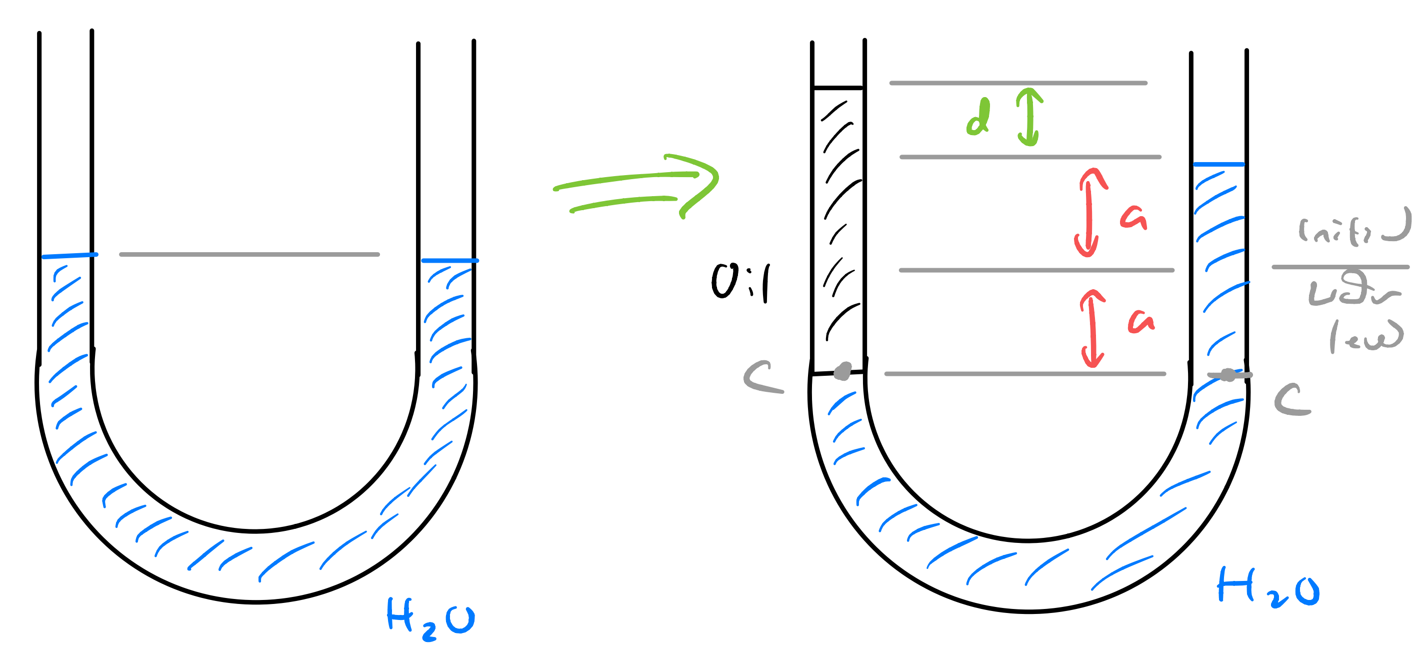
$$= \rho g W \left[\frac{Dy^2}{2} - \frac{y^3}{3} \right|_0^D$$

$$= \rho g W \left(\frac{D^3}{2} - \frac{D^3}{3} \right)$$

$$= \frac{1}{6} \rho g W D^3 \quad \blacksquare$$

Example

A U-tube, in which both ends are open to the atmosphere, is partly filled with water. Oil, which does not mix with water, is poured into one side until it stands a distance $d = 12.3$ mm above the water level on the other side, which has meanwhile risen a distance $a = 67.5$ mm from its original level. Find the density of the oil.



Solution

Points C are at the same pressure.

the pressure change to C from the water side

$$\Delta P = \rho_w g (2a)$$

the pressure drop from Oil side to C

$$\Delta P = \rho_{oil}\, g\, (2a + d)$$

Now, pressure drop to C must be equal

$$\Rightarrow \quad \rho_w g (2a) = \rho_{oil}\, g\, (2a + d)$$

Solve for density of oil

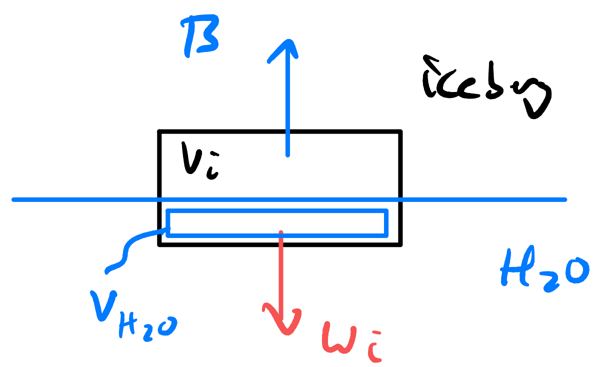
$$\rho_{oil} = \rho_w \frac{2a}{2a + d}$$

$$= \rho_w \frac{1}{1 + \frac{d}{2a}} \simeq 916 \ kg/m^3$$

Example

What fraction of the total volume of an iceberg is exposed?

Solution



Weight of iceberg

$$W_i = \rho_i V_i g$$

Buoyant force

$$B = \rho_{H_2O} V_{H_2O} g$$

↳ volume of water displaced

Density of ice $= 917\ kg/m^3$

Density of seawater $= 1024\ kg/m^3$

Static equilibrium

$$\Rightarrow \sum \vec{F} = 0 \Rightarrow B = W_i$$

or

$$\rho_{H_2O} V_{H_2O} g = \rho_i V_i g$$

$$\Rightarrow \frac{V_{H_2O}}{V_i} = \frac{\rho_i}{\rho_{H_2O}} = 0.896 \Rightarrow 89.6\ \% \ \blacksquare$$

$$\Rightarrow 10.4\% \text{ Exposed} \ \blacksquare$$

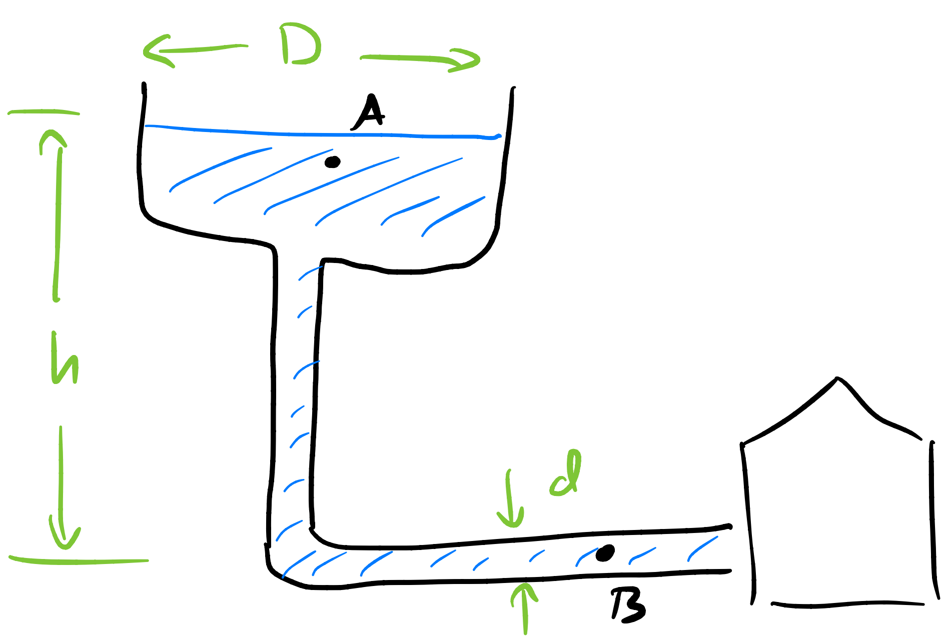
Example

A storage tower of height $h = 32$ m & diameter $D = 3$ m supplies water to a house.

A horizontal pipe at the base of the tower has a diameter $d = 2.54$ cm.

To satisfy the needs of the home, the supply pipe must deliver water at a Rate $R = 0.0025$ m^3/s.

If water were flowing at maximum rate, what is the pressure in the horizontal pipe?



Solution

use Bernoulli's equation between A & B

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g y_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g y_B$$

At point A,

$$P_A = P_o = P_{atmosphere}$$

$$y_A = h$$

So, with $y_B = 0$

$$\Rightarrow P_B = P_o + \rho g h + \frac{1}{2}\rho\left(v_A^2 - v_B^2\right)$$

To get v_A, v_B, use conservation of mass

$$\Rightarrow \frac{dm}{dt} = \rho R = \text{constant}$$

$$= \rho v_A A_A = \rho v_B A_B$$

So, $v_A = \frac{R}{A_A} = \frac{R}{\pi r_A^2} = 3.5 \times 10^{-4}$ m/s

$v_B = \frac{R}{A_B} = \frac{R}{\pi r_B^2} = 4.9$ m/s

See, $v_A \ll v_B$

$$\Rightarrow \quad \frac{1}{2}\rho(v_A^2 - v_B^2) \simeq -\frac{1}{2}\rho v_B^2$$

$$\Rightarrow \quad P = P_0 + \rho g h - \frac{1}{2}\rho v_B^2$$

$P_0 = 1.01 \times 10^5$ Pa

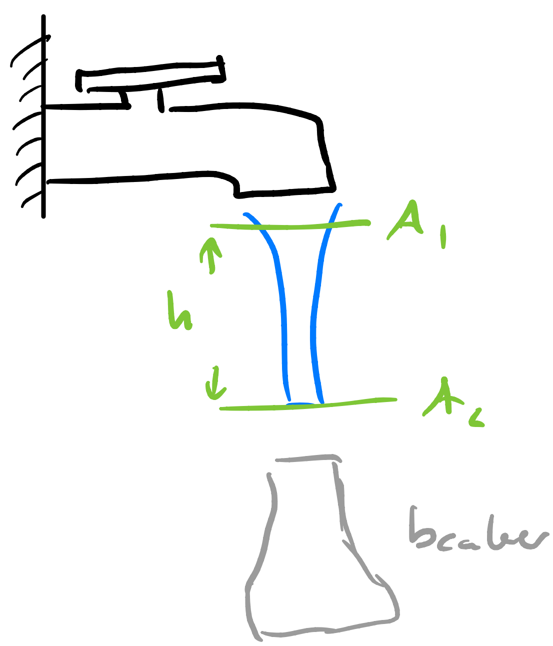
$\rho = 1000$ kg/m^3

$$\Rightarrow \quad P \simeq 4.03 \times 10^5 \text{ Pa} = 4 \text{ atm} \quad \blacksquare$$

Example

Water emerges from a faucet & "necks down" as it falls. The cross-sectional area A_1 is $1.2\ cm^2$, & A_2 is $0.35\ cm^2$. The two levels are separated by a vertical distance $h = 45\ mm$. How long does it take to fill a 100 mL beaker?

Solution



Conservation of mass

$$\frac{dm}{dt} = \rho A v = \text{constant}$$

so,

$$A_1 v_1 = A_2 v_2$$

Apply conservation of energy on fluid element

$$K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \frac{1}{2} m v_2^2 = \frac{1}{2} m v_1^2 + mgh$$

So,

$$v_2^2 = v_1^2 + 2gh$$

Now, solve for v_1

$$v_2 = \frac{A_1}{A_2} v_1$$

$$\Rightarrow \quad v_1 = \sqrt{\frac{2gh\, A_2^2}{A_1^2 - A_2^2}}$$

$$\simeq 0.286 \text{ m/s} \simeq 28.6 \text{ cm/s}$$

So,

$$R = A_1 v_1 = 34 \text{ cm}^3/\text{s}$$

Now, volume of beaker

$$V = RT$$

$$\Rightarrow \quad T = \frac{V}{R} = \frac{100 \text{ mL}}{34 \text{ cm}^3/\text{s}} = \frac{100 \text{ cm}^3}{34 \text{ cm}^3/\text{s}}$$

$$\simeq 2.9 \text{ s}$$