WILLIAM & MARY
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PHYS 303 – Classical Mechanics of Particles and Waves II

Problem Set 5

Due: Thursday, October 17 at 5:00pm

Term: Fall 2024

Instructor: Andrew W. Jackura

Readings

Read sections 9.4–9.10 of Taylor.

Problems

Please indicate the time taken to complete the problem set.

Problem 1. [15 pts.] – Balloon in a Car

A helium balloon is anchored by a massless string to the floor of a car that is accelerating forward with acceleration A. Explain clearly why the balloon tends to tilt forward and find its angle of tilt in equilibrium. *Hint*: Helium balloons float because of the buoyant Archimedean force, which results from a pressure gradient in the air. What is the relation between the directions of the gravitational field and the buoyant force?

Problem 2. [10 pts.] – Levi-Civita Practice I

Consider two vectors, $\mathbf{A} = (1, -2, 4)$ and $\mathbf{B} = (3, 7, -2)$. Find the cross-product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$ using the definition in terms of the Levi-Civita symbol ϵ_{ijk}. Recall that the elements of the vector $\mathbf{C}$ are given by $C_i = \sum_{j,k} \epsilon_{ijk} A_j B_k$.

Problem 3. [10 pts.] – Levi-Civita Practice II

The Levi-Civita symbol satisfies the property

$$\sum_k \epsilon_{ijk}\epsilon_{lmk} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl} \,,$$

where δ_{ij} is the Kronecker symbol. Using this property, prove the vector algebra identity $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{B}\cdot\mathbf{C})(\mathbf{A}\cdot\mathbf{D})$.

Problem 4. [10 pts.] – Pseudo-Forces on Earth

What are the directions of the centrifugal and Coriolis forces on a person moving **(a)** south near the North Pole, **(b)** east on the equator, and **(c)** south across the equator?

Problem 5. [15 pts.] – Coriolis Effect

A bullet of mass m is fired with muzzle speed v_0 horizontally and due north from a position at colatitude θ. Find the direction and magnitude of the Coriolis force in terms of m, v_0, θ, and the earth's angular velocity Ω. How does the Coriolis force compare with the bullet's weight if $v_0 = 1000$ m/s and $\theta = 40°$?

Problem 6. [20 pts.] – Time-Dependent Angular Velocity

Recall the equation of motion for a particle in a rotating frame,

$$m\ddot{\mathbf{r}} = \mathbf{F} + 2m\dot{\mathbf{r}} \times \boldsymbol{\Omega} + m(\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega}\,.$$

The derivation made the assumption that the angular velocity $\boldsymbol{\Omega}$ was constant. Show that if $\dot{\boldsymbol{\Omega}} \neq \mathbf{0}$ then there is a third "fictitious force", sometimes called the *azimuthal force*, on the right hand side equal to $m\mathbf{r} \times \dot{\boldsymbol{\Omega}}$.