PHYS 303 – Classical Mechanics of Particles and Waves II

Problem Set 6

Due: Thursday, October 24 at 5:00pm

Term: Fall 2024

Instructor: Andrew W. Jackura

Readings

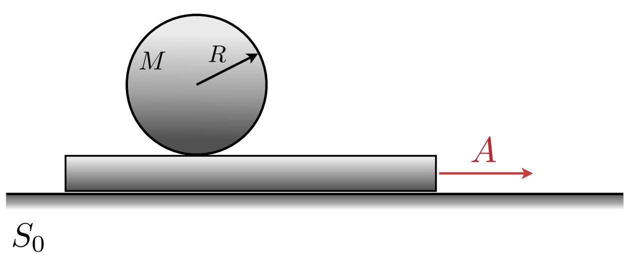
Read sections 10.1–10.5 of Taylor.

Problems

Please indicate the time taken to complete the problem set.

Problem 1. [10 pts.] – Cylinder on Accelerating Plank

A cylinder of radius R and mass M rolls *without slipping* on a plank which is accelerating at a rate A measured in an inertial frame S_0, as shown. Find the acceleration of the cylinder in an inertial frame and in the frame fixed to the plank. Compute the accelerations for two cases, **(a)** a hollow cylinder, and **(b)** a solid cylinder. *Hint*: Recall the moments of inertia for a hollow and solid cylinder, about their center-of-mass, are $I_{\text{hollow}} = MR^2$ and $I_{\text{solid}} = \frac{1}{2}MR^2$.



Problem 2. [25 pts.] – Particle Confined in Rotating Plane

A particle of mass m is confined to move, without friction, in a vertical plane, with axes x horizontal and y vertically up. The plane is forced to rotate with constant angular velocity Ω about the y axis. Find the equations of motion for x and y, solve them, and describe the possible motions.

Problem 3. [25 pts.] – Object Moving Near Earth's Surface I

Using the method of successive approximations, show that if an object is thrown with initial velocity $\mathbf{v}_0$ from a point $\mathcal{O}$ on the earth's surface at colatitude θ, then to first order in Ω (the earth's angular velocity) its trajectory is

$$x = v_{x,0}t + \Omega(v_{y,0}\cos\theta - v_{z,0}\sin\theta)t^2 + \frac{1}{3}\Omega g t^3 \sin\theta + O(\Omega^2)\,,$$

$$y = v_{y,0}t - \Omega(v_{x,0}\cos\theta)t^2 + O(\Omega^2)\,,$$

$$z = v_{z,0}t - \frac{1}{2}g t^2 + \Omega(v_{x,0}\sin\theta)t^2 + O(\Omega^2)\,.$$

Assume that $\mathbf{v}_0$ is small enough to ignore the effects of air drag, and $\mathbf{g}$ remains constant throughout the flight. *Hint*: Recall that the equations of motion are $\ddot{\mathbf{r}} = \mathbf{g} + 2\dot{\mathbf{r}} \times \boldsymbol{\Omega}$, with $\mathbf{g} = \mathbf{g}_0 + (\boldsymbol{\Omega} \times \mathbf{r}) \times \boldsymbol{\Omega}$ the "true" gravitational acceleration. First solve the equations at zeroth order, that is ignoring Ω, then substitute the zeroth-order solution for $\dot{\mathbf{r}}$ on the right-hand side of the equations of motion and integrate to give the first-order in Ω solution.

Problem 4. [25 pts.] – Object Moving Near Earth's Surface II

A baseball is thrown vertically up with initial speed v_0 from a point on the ground at colatitude θ.

(**a**) Using the results of **Problem 3** to show that the ball will return to the ground a distance $(4\Omega v_0^3 \sin\theta)/(3g^2)$ to the west of its launch point.

(**b**) Estimate the size of this effect on the equator if $v_0 = 40$ m/s.

(**c**) Sketch the ball's trajectory as seen from the north (by an observer fixed to the earth). Compare with the orbit of a ball dropped from a point above the equator, and explain why the Coriolis effect moves the dropped ball to the east, but the thrown ball to the west.

Problem 5. [10 pts.] – Center of Mass I

A uniform solid hemisphere of radius R and mass M has its flat base in the xy plane, with its center at the origin. Find the objects center of mass with respect to the origin. *Hint*: Recall that the center of mass for a solid object is $\mathbf{R} = \int \mathbf{r}\,\mathrm{d}m/M$. Write $\mathrm{d}m = \rho\,\mathrm{d}V$ where ρ is the density and $\mathrm{d}V$ is the differential volume. For spherical coordinates, the integral is of the form

$$\int \mathrm{d}V\, f(\mathbf{r}) = \int r^2 \mathrm{d}r \int \sin\theta \mathrm{d}\theta \int \mathrm{d}\phi\, f(r,\theta,\phi)\,.$$

Problem 6. [10 pts.] – Center of Mass II

Find the center of mass of a uniform hemispherical shell of inner and outer radii a and b and mass M positioned with its flat base in the xy plane and its center at the origin.