WILLIAM & MARY
CHARTERED 1693

PHYS 303 – Classical Mechanics of Particles and Waves II

Problem Set 8

Due: Thursday, November 21 at 5:00pm

Term: Fall 2024

Instructor: Andrew W. Jackura

Readings

Read sections 11.1–11.6 and sections 16.1–16.5 of Taylor.

Problems

Please indicate the time taken to complete the problem set.

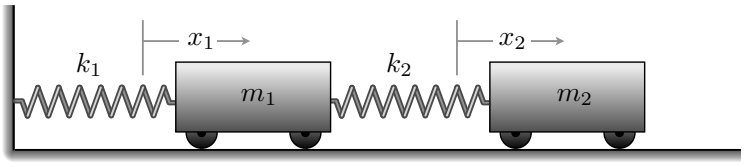
Problem 1. [30 pts.] – Coupled Oscillators I

A cart is connected to the wall and to another cart via a springs, as shown below. Each cart has a mass $m_1 = m_2 = m$ and the springs have spring constant $k_1 = k_2 = k$.

(**a**) **[10 pts.]** Use Newton's laws of motion to derive the equations of motion in terms of the positions x_1 and x_2, which are measured from their respective equilibria.

(**b**) **[10 pts.]** Find the normal mode frequencies, ω_1 and ω_2, for the two carts.

(**c**) **[10 pts.]** Find and describe the motion for each of the normal modes in turn.



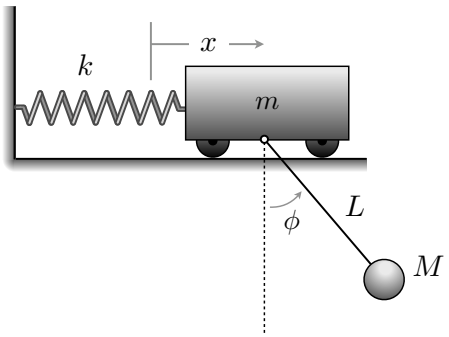
Problem 2. [30 pts.] – Coupled Oscillators II

A simple pendulum (mass M and length L) is suspended from a cart (mass m) that can oscillate on the end of a spring of force constant k, see the figure below.

(**a**) **[10 pts.]** Assuming that both x and ϕ are small, write down the Lagrangian in terms of the two coordinates x and ϕ, where x is the extension of the spring from its equilibrium length and ϕ is the angle of the pendulum measured from its equilibrium position.

(**b**) **[10 pts.]** Find the two Lagrange equations of motion for small oscillations.

(**c**) **[10 pts.]** For $M = m = L = g = 1$ and $k = 2$ (in some units), find the normal frequencies and corresponding normal modes.



Problem 3. [20 pts.] – Another Double Pendulum

Consider two identical plane pendulums (each of length L and mass m) that are joined by a massless spring (force constant k) The pendulums' positions are specified by the angles ϕ_1 and ϕ_2 as shown in the figure below. The natural length of the spring is equal to the distance between the two supports, so the equilibrium position is at $\phi_1 = \phi_2 = 0$ with the two pendulums vertical.

(**a**) **[10 pts.]** Assuming the angles remain small, write down the Lagrangian and the corresponding Euler-Lagrange equations of motion. Small angles means that the extension of the spring is well approximated by $L(\phi_2 - \phi_1)$.

(**b**) **[10 pts.]** Find and describe the normal modes for these two coupled pendulums.