WILLIAM & MARY
CHARTERED 1693

PHYS 303 – Classical Mechanics of Particles and Waves II

Problem Set 9

Due: Thursday, December 5 at 5:00pm

Term: Fall 2024

Instructor: Andrew W. Jackura

Readings

Read sections 16.1–16.11 of Taylor.

Problems

Please indicate the time taken to complete the problem set.

Problem 1. [10 pts.] – Wave Speed

Verify that the quantity $c = \sqrt{T/\mu}$ that appears in the wave equation for a string does indeed have the units of a speed.

Problem 2. [20 pts.] – Wave on a String

A taut string of length $L = 1$ is released from rest at $t = 0$, with initial position

$$u(x,0) = \begin{cases} 2x & 0 \leq x < \frac{1}{2} \\ 2(1-x) & \frac{1}{2} \leq x \leq 1 \end{cases} .$$

Take the wave speed on the string to be $c = 1$. Following the procedure in lecture, write down the most general solution of the wave equation,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} ,$$

subject to the boundary and initial conditions. *Hint:* You solutions should be of the form $u(x,t) = \sum_n B_n \sin(k_n x)\cos(\omega_n t)$, where B_n is determined from initial conditions and k_n and ω_n are to be fixed from the given parameters and boundary conditions.

Problem 3. [20 pts.] – Stress Tensor

It is found that the stress tensor at any point (x, y, z) in a certain continuous medium has the form (with an unspecified, convenient choice of units)

$$\boldsymbol{\Sigma} = \begin{pmatrix} xz & z^2 & 0 \\ z^2 & 0 & -y \\ 0 & -y & 0 \end{pmatrix} .$$

Find the surface force on a small area $\mathrm{d}A$ of the surface $x^2 + y^2 + 2z^2 = 4$ at the point $(1, 1, 1)$. Find the angle between the unit normal of the small area, $\hat{\mathbf{n}}$, and the resulting surface force.