Feynman Rules - Electroweak Model of Leptons

The Lagrangian density for the leptonic electroweak (EW) model, after spontaneous symmetry breaking in the *unitary gauge*, is given by

$$
\begin{aligned}
\mathcal{L}_{\text{ew}} = & \frac{i}{2}\sum_A \bar{\ell}_A \overleftrightarrow{\partial\!\!\!/} \ell_A + \frac{i}{2}\sum_A \bar{\nu}_A \overleftrightarrow{\partial\!\!\!/} \nu_A + \text{h.c.} + \frac{1}{2}\partial_\mu h \partial^\mu h \\
& - \frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu - ig\cos\theta_W (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)\right)^2 \\
& - \frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - ie (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)\right)^2 \\
& - \frac{1}{2}\left|\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+ + ig\cos\theta_W (W_\mu^+ Z_\nu - W_\nu^+ Z_\mu) + ie(W_\mu^+ A_\nu - W_\nu^+ A_\mu)\right|^2 \\
& - \sum_A m_{\ell_A}\bar{\ell}_A \ell_A - \frac{1}{2}m_h^2 h^2 + m_W^2 W_\mu^+ W^{-\,\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\
& - \frac{g}{\sqrt{2}}\bar{\nu}_A \gamma^\mu \frac{1}{2}(1-\gamma_5)\ell_A\, W_\mu^+ + \text{h.c.} - Qe\sum_A \bar{\ell}_A \gamma^\mu \ell_A\, A_\mu \\
& - \frac{g}{\cos\theta_W}\sum_A \bar{\ell}_A \gamma^\mu \left(-\frac{1}{4} + \sin^2\theta_W + \frac{1}{4}\gamma_5\right)\ell_A\, Z_\mu - \frac{g}{\cos\theta_W}\sum_A \bar{\nu}_A \gamma^\mu \left(\frac{1}{4} - \frac{1}{4}\gamma_5\right)\nu_A\, Z_\mu^0 \\
& - \frac{1}{2}g\sum_A \frac{m_{\ell_A}}{m_W} h \bar{\ell}_A \ell_A - \frac{3}{2}g\frac{m_h^2}{m_W}\, h^3 - \frac{3}{4}g^2 \frac{m_h^2}{m_W^2}\, h^4 \\
& + \frac{1}{4}g^2 h^2 W_\mu^+ W^{-\,\mu} + \frac{1}{8}\frac{g^2}{\cos^2\theta_W} h^2 Z_\mu Z^\mu + m_W g h W_\mu^+ W^{-\,\mu} + \frac{1}{2}\frac{g}{\cos\theta_W} m_W\, h Z_\mu Z^\mu
\end{aligned}
\tag{1}
$$

where $A = \{e, \mu, \tau\}$ are the leptons, $Q = -e$ with e. Here the chosen parameters are $\cos\theta_W = \sqrt{1 - \sin^2\theta_W}$ with $\sin^2\theta_W \approx 0.23$, $e \approx 0.303$, $g \approx 0.63$, with all others given by the masses of the fermions and the Higgs boson.

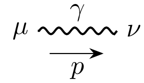
Feynman Rules

Here we give the Feynman rules for the scattering amplitude $\mathcal{M}$,

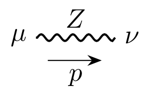
$$i\mathcal{M} = \text{sum of all connected, amputated diagrams}\,,$$

where the diagrams are evaluated according to the following rules:

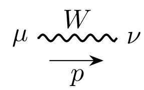
- Draw all topologically distinct diagrams at a given order;
- For each internal gauge boson line, attach a propagator



$$= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon}\,;$$

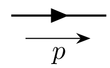


$$= \frac{-i}{p^2 - m_Z^2 + i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_Z^2}\right);$$



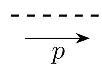
$$= \frac{-i}{p^2 - m_W^2 + i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2}\right);$$

- For each internal fermion line, attach a propagator



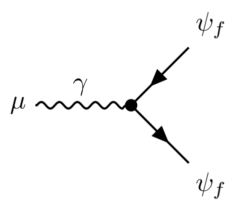
$$= \frac{i(\not{p} + m_f)}{p^2 - m_f^2 + i\epsilon};$$

- For each internal higgs boson line, attach a propagator

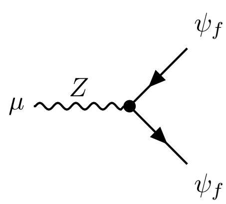


$$= \frac{i}{p^2 - m_h^2 + i\epsilon};$$

- For each fermion-neutral gauge boson vertex, assign



$$= -iQ_f e\gamma^\mu;$$

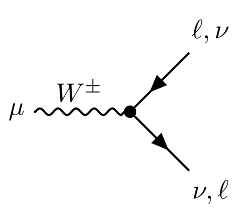


$$= -i\frac{g}{\cos\theta_W}\gamma^\mu(v_f - a_f\gamma_5);$$

$$v_f = \frac{1}{2}T_f^3 - Q_f\sin^2\theta_W\,, \qquad a_f = \frac{1}{2}T_f^3\,.$$

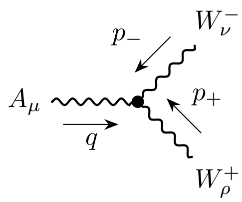
Note: $T_\ell^3 = -1/2$, $T_\nu^3 = +1/2$, $Q_\ell = -1$, $Q_\nu = 0$.

- For each fermion-charged gauge boson vertex, assign

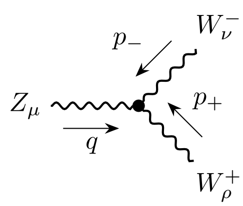


$$= -i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1}{2}(1-\gamma_5);$$

- For each triple gauge vertex, assign

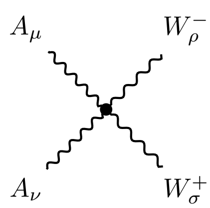


$$= ie\left[(q-p_-)^\rho g^{\mu\nu} + (p_- - p_+)^\mu g^{\nu\rho} + (p_+ - q)^\nu g^{\mu\rho}\right];$$

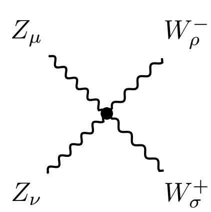


$$= -ig\cos\theta_W\left[(q-p_-)^\rho g^{\mu\nu} + (p_- - p_+)^\mu g^{\nu\rho} + (p_+ - q)^\nu g^{\mu\rho}\right];$$

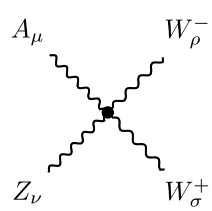
- For each quartic gauge vertex, assign



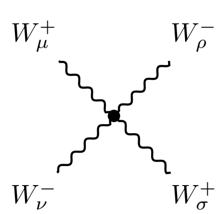
$$= -ie^2\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}\right];$$



$$= -ig^2\cos^2\theta_W\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}\right];$$

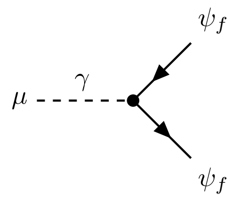


$$= -ieg\cos\theta_W\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}\right];$$



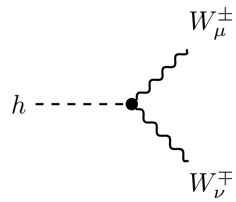
$$= ig^2\left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}\right];$$

- For each fermion-Higgs vertex, assign

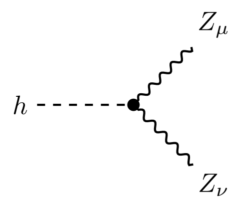


$$= -i\frac{g}{2}\frac{m_f}{m_W}\,;$$

- For each gauge-Higgs three-vertex, assign

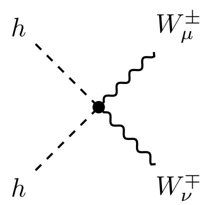


$$= igm_W g^{\mu\nu}\,;$$

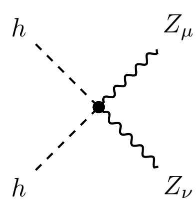


$$= i\frac{g}{\cos\theta_W}m_Z g^{\mu\nu}\,;$$

- For each gauge-Higgs four-vertex, assign

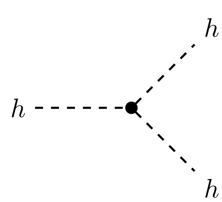


$$= -i\frac{g^2}{2}g^{\mu\nu}\,;$$

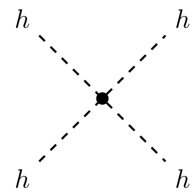


$$= -i\frac{1}{2}\frac{g^2}{\cos^2\theta_W}g^{\mu\nu}\,;$$

- For each Higgs self-interaction, assign

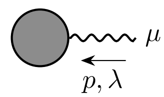


$$= -i\frac{3}{2}g\frac{m_h^2}{m_W}\,;$$

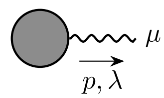


$$= -i\frac{3}{4}g^2\frac{m_h^2}{m_W^2}\,;$$

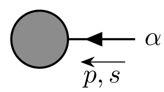
- For each external line, place the particle on the "mass-shell", $p^2 = m_q^2$ for the quark and $p^2 = 0$ for the gluon, and attach a wavefunction factor

"incoming boson" 

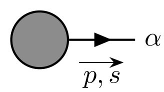
$= \epsilon_\mu(p,\lambda)\,;$

"outgoing boson" 

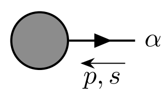
$= \epsilon^*_\mu(p,\lambda)\,;$

"incoming lepton" 

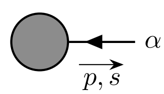
$= u_\alpha(p,s)\,;$

"outgoing lepton" 

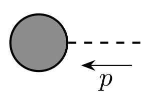
$= \bar{u}_\alpha(p,s)\,;$

"incoming anti-lepton" 

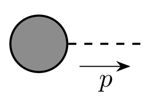
$= \bar{v}_\alpha(p,s)\,;$

"outgoing anti-lepton" 

$= v_\alpha(p,s)\,;$

"incoming Higgs" 

$= 1\,;$

"outgoing Higgs" 

$= 1\,;$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\int \frac{\mathrm{d}^4k}{(2\pi)^4}$;
- For each fermion loop, multiply the diagram by (-1);
- For each set of diagram which are only distinguished by interchanging two external fermion lines, multiply one of the diagrams by (-1);
- Multiply the contribution for each diagram by an appropriate symmetry factor $\mathcal{S}^{-1}$ for identical particles.