Feynman Rules - Quantum Chromodynamics

The Lagrangian density for Quantum Chromodynamics (QCD) is given by

$$\mathcal{L} = \frac{1}{2}\sum_f i\bar{\boldsymbol{\psi}}_f \not{\mathbf{D}}\boldsymbol{\psi}_f + \text{h.c.} - \sum_f m_f \bar{\boldsymbol{\psi}}_f\boldsymbol{\psi}_f - \frac{1}{2}\mathrm{tr}(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}) - \frac{1}{2\xi}(\partial^\mu \mathbf{A}_\mu)^2 - \partial_\mu \bar{\mathbf{c}}\mathbf{D}^\mu\mathbf{c} \tag{1}$$

where the gauge covariant derivative is $\mathbf{D}_\mu = \mathbf{1}\partial_\mu + ig_s\mathbf{A}_\mu$ and the field strength tensor is $\mathbf{G}_{\mu\nu} = \partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu$. Here m is the mass of the charged fermion. We use indices α, β for spinor space, while μ, ν for Lorentz space. Also, j, k indicate the color indices in the $\mathbf{3}$ representation of $\mathrm{SU}(3)_c$, and a, b are the indices in the $\mathbf{8}$ of $\mathrm{SU}(3)_c$. Therefore, the components of the fermion field are $(\psi_f)^\alpha_j$, for the gluon field $(A_\mu)_{jk} = A^a_\mu(T_a)_{jk}$ where $T_a = \lambda_a/2$, and for the ghost field c_a.

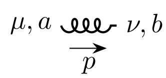
Feynman Rules

Here we give the Feynman rules for the scattering amplitude $\mathcal{M}$,

$$i\mathcal{M} = \text{sum of all connected, amputated diagrams}\,,$$

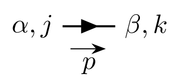
where the diagrams are evaluated according to the following rules:

- Draw all topologically distinct diagrams at a given order;
- For each internal gluon line, attach a propagator



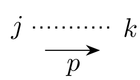
$$= \frac{-i\delta_{ab}}{p^2 + i\epsilon}\left(g_{\mu\nu} - (1-\xi)\frac{p_\mu p_\nu}{p^2}\right)\,;$$

- For each internal quark line, attach a propagator



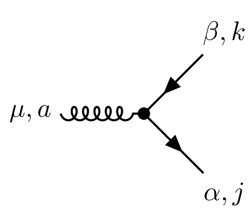
$$= \delta_{jk}\,\frac{i(\not{p} + m_f)_{\beta\alpha}}{p^2 - m_f^2 + i\epsilon}\,;$$

- For each internal ghost line, attach a propagator



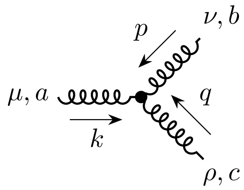
$$= \delta_{jk}\,\frac{-i}{p^2 + i\epsilon}\,;$$

- For each quark-gluon vertex, assign



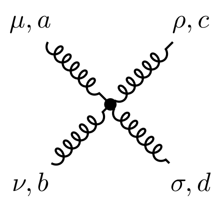
$$= -ig_s(\gamma^\mu)_{\beta\alpha}(T_a)_{jk}\,;$$

- For each three-gluon vertex, assign



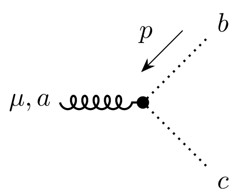
$$= -ig_s(-if_{abc})\Big[(k-p)^\rho g^{\mu\nu} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\mu\rho}\Big]\,;$$

- For each four-gluon vertex, assign



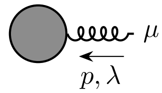
$$\begin{aligned} = -ig_s^2\Big[\quad & f_{abe}f_{cde}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) \\ & + f_{ace}f_{bde}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}) \\ & + f_{ade}f_{bce}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma})\Big]\,; \end{aligned}$$

- For each ghost-gluon vertex, assign

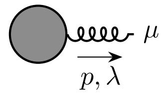


$$= -ig_s(-if_{abc})p^\mu\,;$$

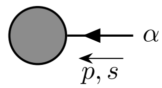
- For each external line, place the particle on the "mass-shell", $p^2 = m_q^2$ for the quark and $p^2 = 0$ for the gluon, and attach a wavefunction factor

"incoming gluon"


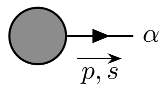
$= \epsilon_\mu(p, \lambda)\,;$

"outgoing gluon"


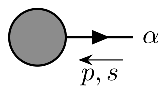
$= \epsilon^*_\mu(p, \lambda)\,;$

"incoming quark"


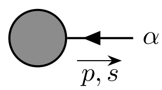
$= u_\alpha(p, s)\,;$

"outgoing quark"


$= \bar{u}_\alpha(p, s)\,;$

"incoming anti-quark"


$= \bar{v}_\alpha(p, s)\,;$

"outgoing anti-quark"


$= v_\alpha(p, s)\,;$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\int \frac{\mathrm{d}^4k}{(2\pi)^4}$;
- For each fermion loop, multiply the diagram by (-1);
- For each set of diagram which are only distinguished by interchanging two external fermion lines, multiply one of the diagrams by (-1);