Feynman Rules - Self-Interacting Scalar theory

The Lagrangian density for a self-interacting real-scalar field theory is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 - \frac{1}{4!}\lambda\varphi^4$$

where m is the mass parameter and λ is the quartic coupling.

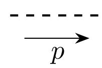
Feynman Rules

Here we give the Feynman rules for the scattering amplitude $\mathcal{M}$,

$$i\mathcal{M} = \text{sum of all connected, amputated diagrams}\,,$$

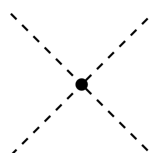
where the diagrams are evaluated according to the following rules:

- Draw all topologically distinct diagrams at a given order;
- For each internal scalar line, attach a propagator



$$= \frac{i}{p^2 - m^2 + i\epsilon}\,;$$

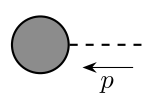
- For each vertex, assign



$$= -i\lambda\,;$$

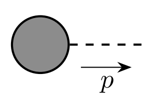
- For each external scalar line, place on the mass-shell $p^2 = m^2$ and attach a wavefunction factor

"incoming scalar"



$$= 1\,;$$

"outgoing scalar"



$$= 1\,;$$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\int \frac{\mathrm{d}^4k}{(2\pi)^4}$;
- Multiply the contribution for each diagram by an appropriate symmetry factor $\mathcal{S}^{-1}$.
- For scattering amplitudes, place all external lines on their mass-shell $p^2 = m^2$ and multiply by the scalar wavefunction "1".