Feynman Rules - The Standard Model

The Lagrangian density for the Standard Model, after spontaneous symmetry breaking in the *unitary gauge* for the EW part and ignoring the gauge fixing for QCD, is given by

$$
\begin{aligned}
\mathcal{L}_{\rm sm} = & \frac{i}{2}\sum_A \bar{u}_A \overleftrightarrow{\partial\!\!\!/} u_A + \frac{i}{2}\sum_A \bar{d}_A \overleftrightarrow{\partial\!\!\!/} d_A + \frac{i}{2}\sum_A \bar{\ell}_A \overleftrightarrow{\partial\!\!\!/} \ell_A + \frac{i}{2}\sum_A \bar{\nu}_A \overleftrightarrow{\partial\!\!\!/} \nu_A + \text{h.c.} + \frac{1}{2}\partial_\mu h \partial^\mu h \\
& - \frac{1}{4}\left(\partial_\mu Z_\nu - \partial_\nu Z_\mu - ig\cos\theta_W (W^+_\mu W^-_\nu - W^-_\mu W^+_\nu)\right)^2 \\
& - \frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu)\right)^2 \\
& - \frac{1}{2}\left|\partial_\mu W^+_\nu - \partial_\nu W^+_\mu + ig\cos\theta_W (W^+_\mu Z_\nu - W^+_\nu Z_\mu) + ie(W^+_\mu A_\nu - W^+_\nu A_\mu)\right|^2 \\
& - \sum_A m_{u_A} \bar{u}_A u_A - \sum_A m_{d_A} \bar{d}_A d_A - \sum_A m_{\ell_A} \bar{\ell}_A \ell_A - \frac{1}{2} m_h^2 h^2 + m_W^2 W^+_\mu W^{-\,\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \\
& - \frac{g}{\sqrt{2}} \sum_A \bar{\nu}_A \gamma^\mu P_L \ell_A \, W^+_\mu + \text{h.c.} - \frac{g}{\sqrt{2}} \sum_{A,B} \bar{d}_A \gamma^\mu P_L V_{AB} u_B \, W^+_\mu + \text{h.c.} \\
& - \sum_f Q_f e \bar{f} \gamma^\mu f \, A_\mu - \frac{g}{\cos\theta_W} \sum_f \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f \, Z_\mu \\
& - \frac{1}{2} g \sum_A \frac{m_{u_A}}{m_W} h \bar{u}_A u_A - \frac{1}{2} g \sum_A \frac{m_{d_A}}{m_W} h \bar{d}_A d_A - \frac{1}{2} g \sum_A \frac{m_{\ell_A}}{m_W} h \bar{\ell}_A \ell_A - \frac{3}{2} g \frac{m_h^2}{m_W} h^3 - \frac{3}{4} g^2 \frac{m_h^2}{m_W^2} h^4 \\
& + \frac{1}{4} g^2 h^2 W^+_\mu W^{-\,\mu} + \frac{1}{8} \frac{g^2}{\cos^2\theta_W} h^2 Z_\mu Z^\mu + m_W g h W^+_\mu W^{-\,\mu} + \frac{1}{2} \frac{g}{\cos\theta_W} m_W \, h Z_\mu Z^\mu \\
& - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\,a}
\end{aligned}
\tag{1}
$$

where A is the generation index $A = \{1, 2, 3\}$, ℓ_A are the leptons $\ell_A = \{e, \mu, \tau\}$, ν_A are the neutrinos $\nu_A = \{\nu_e, \nu_\mu, \nu_\tau\}$, u_A are the up-type quarks $u_A = \{u, c, t\}$, d_A are the down-type quarks $d_A = \{d, s, b\}$, f is the entire set of fermions.

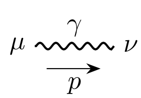
Feynman Rules

Here we give the Feynman rules for the scattering amplitude $\mathcal{M}$,

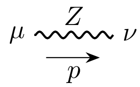
$$i\mathcal{M} = \text{sum of all connected, amputated diagrams}\,,$$

where the diagrams are evaluated according to the following rules:

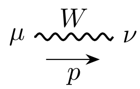
- Draw all topologically distinct diagrams at a given order;
- For each internal gauge boson line, attach a propagator



$$= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon}\,;$$

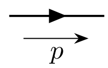


$$= \frac{-i}{p^2 - m_Z^2 + i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_Z^2}\right) ;$$



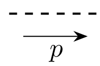
$$= \frac{-i}{p^2 - m_W^2 + i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_W^2}\right) ;$$

- For each internal fermion line, attach a propagator



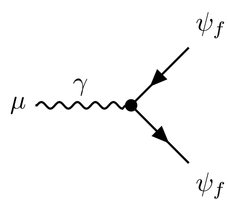
$$= \frac{i(\not{p} + m_f)}{p^2 - m_f^2 + i\epsilon} ;$$

- For each internal higgs boson line, attach a propagator

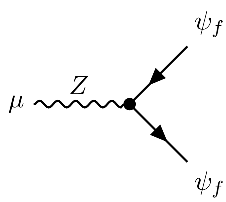


$$= \frac{i}{p^2 - m_h^2 + i\epsilon} ;$$

- For each fermion-neutral gauge boson vertex, assign



$$= -iQ_f e\gamma^\mu ;$$

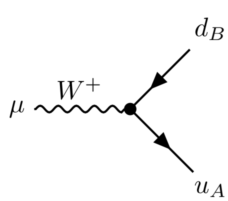


$$= -i\frac{g}{\cos\theta_W}\gamma^\mu(v_f - a_f\gamma_5) ;$$

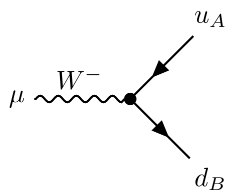
$$v_f = \frac{1}{2}T_f^3 - Q_f\sin^2\theta_W , \qquad a_f = \frac{1}{2}T_f^3 .$$

Note: $T_\ell^3 = -1/2$, $T_\nu^3 = +1/2$, $Q_\ell = -1$, $Q_\nu = 0$.

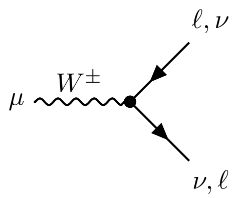
- For each fermion-charged gauge boson vertex, assign



$$= -i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1}{2}(1 - \gamma_5)V_{AB} ;$$

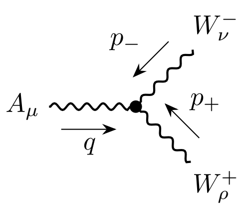


$$= -i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1}{2}(1-\gamma_5)V^*_{AB}\,;$$

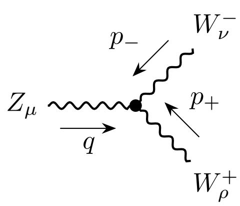


$$= -i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1}{2}(1-\gamma_5)\,;$$

- For each triple gauge vertex, assign

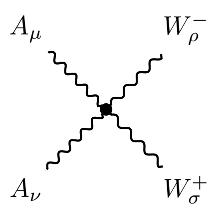


$$= ie\Big[(q-p_-)^\rho g^{\mu\nu} + (p_- - p_+)^\mu g^{\nu\rho} + (p_+ - q)^\nu g^{\mu\rho}\Big]\,;$$

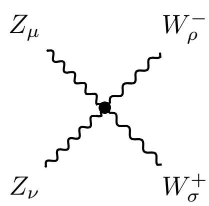


$$= -ig\cos\theta_W\Big[(q-p_-)^\rho g^{\mu\nu} + (p_- - p_+)^\mu g^{\nu\rho} + (p_+ - q)^\nu g^{\mu\rho}\Big]\,;$$

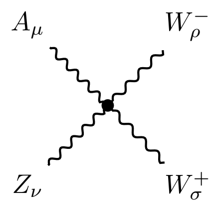
- For each quartic gauge vertex, assign



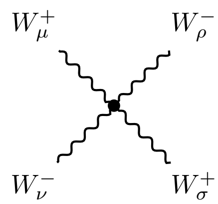
$$= -ie^2\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}\Big]\,;$$



$$= -ig^2\cos^2\theta_W\Big[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}\Big]\,;$$

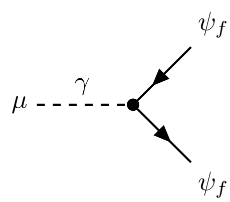


$$= -ieg\cos\theta_W \left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma} \right];$$



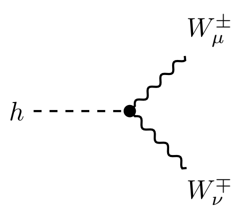
$$= ig^2 \left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma} \right];$$

- For each fermion-Higgs vertex, assign

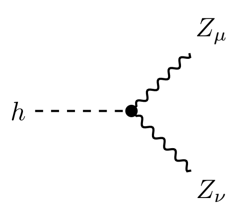


$$= -i\frac{g}{2}\frac{m_f}{m_W};$$

- For each gauge-Higgs three-vertex, assign

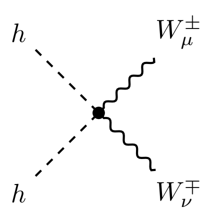


$$= igm_W g^{\mu\nu};$$

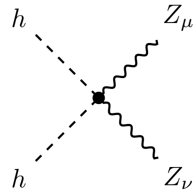


$$= i\frac{g}{\cos\theta_W}m_Z g^{\mu\nu};$$

- For each gauge-Higgs four-vertex, assign

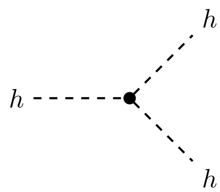


$$= -i\frac{g^2}{2}g^{\mu\nu};$$

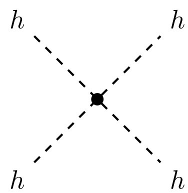


$$= -i\frac{1}{2}\frac{g^2}{\cos^2\theta_W}g^{\mu\nu}\,;$$

- For each Higgs self-interaction, assign

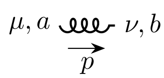


$$= -i\frac{3}{2}g\frac{m_h^2}{m_W}\,;$$



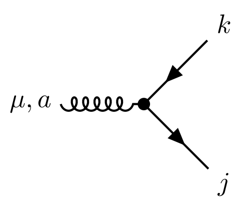
$$= -i\frac{3}{4}g^2\frac{m_h^2}{m_W^2}\,;$$

- For each internal gluon line, attach a propagator



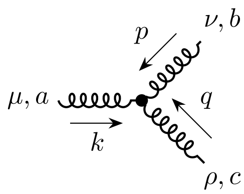
$$= \frac{-i\delta_{ab}}{p^2+i\epsilon}\left(g_{\mu\nu} - (1-\xi_G)\frac{p_\mu p_\nu}{p^2}\right)\,;$$

- For each quark-gluon vertex, assign



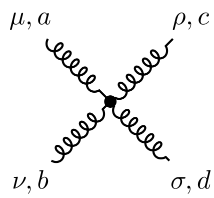
$$= -ig_s\gamma^\mu\,(T_a)_{jk}\,;$$

- For each three-gluon vertex, assign



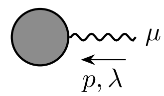
$$= -ig_s(-if_{abc})\Big[(k-p)^\rho g^{\mu\nu} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\mu\rho}\Big]\,;$$

- For each four-gluon vertex, assign

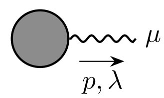


$$= -ig_s^2 \Big[f_{abe} f_{cde} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f_{ace} f_{bde} (g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f_{ade} f_{bce} (g_{\mu\nu} g_{\rho\sigma} - g_{\mu\rho} g_{\nu\sigma}) \Big] \,;$$

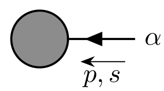
- For each external line, place the particle on the "mass-shell", $p^2 = m_q^2$ for the quark and $p^2 = 0$ for the gluon, and attach a wavefunction factor

"incoming boson" 

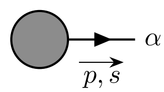
$= \epsilon_\mu(p, \lambda)\,;$

"outgoing boson" 

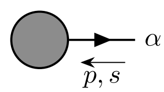
$= \epsilon^*_\mu(p, \lambda)\,;$

"incoming lepton" 

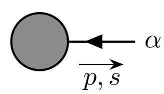
$= u_\alpha(p, s)\,;$

"outgoing lepton" 

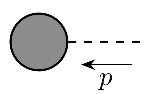
$= \bar{u}_\alpha(p, s)\,;$

"incoming anti-lepton" 

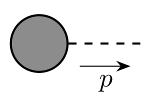
$= \bar{v}_\alpha(p, s)\,;$

"outgoing anti-lepton" 

$= v_\alpha(p, s)\,;$

"incoming Higgs" 

$= 1\,;$

"outgoing Higgs" 

$= 1\,;$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\int \frac{\mathrm{d}^4 k}{(2\pi)^4}$;
- For each fermion loop, multiply the diagram by (-1);

- For each set of diagram which are only distinguished by interchanging two external fermion lines, multiply one of the diagrams by (-1);
- Multiply the contribution for each diagram by an appropriate symmetry factor $\mathcal{S}^{-1}$ for identical particles.