Leptonic Electroweak Model

Let's construct the gauge theory of weak interactions. This is a "simple" model of leptons, Higgs, and EW bosons. The model is based on the gauge field theory with product group $SU(2)_L \times U(1)_Y$

generators of $su(2)_L$: T^a , $a = 1,2,3$

e.g., for $\underset{\sim}{2}$, $T^a = \frac{1}{2}\sigma^a$ weak isospin

generators of $u(1)_Y$: Y weak hypercharge

the algebra is $su(2)_L \oplus u(1)_Y$

$$\boxed{\begin{aligned} [T^a, T^b] &= i\epsilon^{abc}T^c \\ [Y, Y] &= 0 \\ [T^a, Y] &= 0 \end{aligned}}$$

We can define the electric charge generator Q that generates the $U(1)_Q$ of QED. This is given by "Gell-Man – Nishijima relation"

$$\boxed{Q = T^3 + \frac{1}{2}Y}$$

Not related to strong interaction version

The algebra is then

$$[Q,Q]=0$$
$$[Y,Q]=0$$
$$[T^a,Q]=[T^a,T^3]=i\epsilon^{a3c}T^c$$

$$\Rightarrow\quad [T^3,Q]=0\,,\quad [T^1,Q]=-iT^2\,,\quad [T^2,Q]=iT^1$$

Basic particles in leptonic EW model

1. spin-$\frac{1}{2}$ leptons : $e,\ \nu_e,\ \mu,\ \nu_\mu,\ \tau,\ \nu_\tau$
2. spin-1 gauge bosons : $A_\mu,\ W_\mu^\pm,\ Z_\mu^0$
3. spin-0 Higgs boson : h

No quarks or strong interactions (will add later)

In 1957, Wu et al. found electrons emitted in the nuclear β-decay of $^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$, in a preffered direction opposite to the nuclear spin.
Can conclude that parity is violated in weak processes.
Observe that parity is maximally violated, i.e., left-handed particles experience weak interaction, but not right-handed.
$\Rightarrow$ Matter fields must be constructed to respect this.

① Spinor fields for leptons labeled by "generation"

Charged leptons : $\ell_A = (e, \mu, \tau)$, $A = 1,2,3 \sim e, \mu, \tau$

neutrino fields : $\nu_A = (\nu_e, \nu_\mu, \nu_\tau)$

Introduce left-and-right-handed fields,

$$\psi_{\substack{L\\R}} = \frac{1}{2}(1 \mp \gamma_5)\psi .$$

For each family ℓ_A, ν_A in generation.

Define

$$L_A = \begin{pmatrix} \nu_{LA} \\ \ell_{LA} \end{pmatrix} , \quad R_A = \ell_{RA}$$

will be an $SU(2)_L$ singlet

will be a $SU(2)_L$ doublet

For basic EW model, no right-handed neutrinos

$\Rightarrow$ No ν_{RA} fields $\Rightarrow$ massless neutrinos

② Vector fields are all gauge fields

$$SU(2)_L \otimes U(1)_Y$$

3 gens. 1 gen. $\Rightarrow$ 4 gauge fields

W_μ^a $\quad$ B_μ

W_μ^i is an $SU(2)_L$ triplet. B_μ is a $U(1)_Y$ singlet.

Useful combinations: $$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm i W_\mu^2)$$
$$W_\mu^- = (W_\mu^+)^*$$

We will see later that A_μ & Z_μ^0 are linear combos of W_μ^3 & B_μ.

(3) Scalar fields are Higgs fields.
EW model has 4 real Higgs fields to give mass to $W_\mu^\pm$, Z_μ^0. Write these as 2 complex scalars

$$\phi^+ , \phi^0$$

and define complex conjugates

$$\phi^- = (\phi^+)^* , \phi^{0*}$$

Useful to define $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, which is an $SU(2)_L$ doublet.

$$\Rightarrow \phi^\dagger = (\phi^-, \phi^{0*}) = \phi^{*T}$$

& $$\phi^c = i\sigma^2 \phi^* = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi^- \\ \phi^{0*} \end{pmatrix} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} \in \underset{\sim}{2} \text{ of } SU(2)_L$$

Warning: $\overline{\psi}_L$ is understood as $\overline{(\psi_L)}$, which is right-handed! (see HW)

Representations & Fields

Field	$SU(2)_L \; U(1)_Y$	$SU(2)_L : T$	$SU(2)_L : T_3$	$U(1)_Y : Y$	$U(1)_Q : Q$
$L_A = \begin{pmatrix} \nu_A \\ \ell_A \end{pmatrix}_L$	$\underset{\sim}{2}_{-1}$	$\frac{1}{2}$ $\frac{1}{2}$	$+\frac{1}{2}$ $-\frac{1}{2}$	-1 -1	0 -1
$R_A = \ell_{R_A}$	$\underset{\sim}{1}_{-2}$	0	0	-2	-1
$W_\mu = \begin{pmatrix} W_\mu^+ \\ W_\mu^0 \\ W_\mu^- \end{pmatrix}$	$\underset{\sim}{3}_0$	1 1 1	$+1$ 0 -1	0 0 0	$+1$ 0 -1
B_μ	$\underset{\sim}{1}_0$	0	0	0	0
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\underset{\sim}{2}_{+1}$	$1/2$ $1/2$	$+1/2$ $-1/2$	$+1$ $+1$	$+1$ 0

The covariant derivative is

Standard convention

$$(D_\mu)_{jk} = \delta_{jk}\,\partial_\mu + i g W_\mu^{\,a} (T_a)_{jk} + i\left(\tfrac{1}{2}g'\right) B_\mu Y \delta_{jk}$$

$SU(2)_L$ coupling

$U(1)_Y$ coupling

Generators are for following multiplets

Multiplet	$(T^a)_{jk}$	Y
L_1	$\frac{1}{2}(\sigma^a)_{jk}$	-1
R_1	0	-2
ϕ	$\frac{1}{2}(\sigma^a)_{jk}$	-1

The gauge field strengths are

$$\boxed{\begin{aligned} W_{\mu\nu}^{\,a} &= \partial_\mu W_\nu^{\,a} - \partial_\nu W_\mu^{\,a} - g\,\epsilon^{abc} W_\mu^{\,b} W_\nu^{\,c} \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned}}$$

The leptonic EW model is then given by

$$\boxed{\begin{aligned}\mathcal{L} = & -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + \frac{1}{2} i \bar{L}_A \not{D} L_A + \frac{1}{2} i \bar{R}_A \not{D} R_A + h.c. \\ & + (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!} (\phi^\dagger \phi)^2 \\ & - G^L_A [\bar{R}_A (\phi^\dagger L_A) + (\bar{L}_A \phi) R_A]\end{aligned}}$$

Gauge KE

Fermion KE

Higgs KE
+ unstable potential

Yukawa coupling
(sum on A)

All terms are individually $SU(2)_L \otimes U(1)_Y$ invariant.

Note that D_μ means different things acting on different fields. Note that $\phi^\dagger L_A$ is understood as product $(\phi^-, \phi^{0*}) \begin{pmatrix} \nu_{L_A} \\ \ell_{L_A} \end{pmatrix}$ in $SU(2)_L$ space.

This is (almost) the most general $\mathcal{L}$. Notice that there are no mass terms for fermions (would break chiral $SU(2)_L$). Also, notice that G^L_A could be a matrix in general. For this model, we can diagonalize. Later in SM, we will see mass mixing.

The number of free parameters is 7!

$$g,\ g',\ \mu^2,\ \lambda,\ G_e^L,\ G_\mu^L,\ G_\tau^L$$

These need to be measured in order to make predictions. The potential for the Higgs is most general for this model, since $\phi^{c\dagger}\phi^c = \phi^\dagger\phi$. Also, No Majorana mass terms are compatible for ν are compatible with $SU(2)_L \otimes U(1)_Y$. Turns out to be tricky to add ν masses (see later).

The Higgs potential induces SSB.
To see what happens write ϕ as

$$\underbrace{\phi(x)}_{\text{4 real fields}} = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\Big(\tfrac{1}{2} i \underbrace{\theta^a(x)}_{\text{3 real fields}} \sigma^a \Big) \begin{pmatrix} 0 \\ \underbrace{r(x)}_{\text{1 real field}} \end{pmatrix}$$

Next, choose unitary gauge to interpret, where $SU(2)_L$ gauge parameters are $\alpha^a(x) = -\theta^a(x)$

$\Rightarrow$ 3 of 4 components of Higgs "disappear"

So, unitary gauge $\Rightarrow \phi \to \exp\left(-\frac{1}{2} i \theta^a(x) \sigma^a\right)\phi$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ r \end{pmatrix}$$

So,

$$\mathcal{L}_{Higgs} = \frac{1}{2}\left[D_\mu \begin{pmatrix} 0 \\ r \end{pmatrix}\right]^\dagger \left[D^\mu \begin{pmatrix} 0 \\ r \end{pmatrix}\right] - U(r)$$

where $U(r) = -\frac{\lambda}{4!}(r^2 - a^2)^2$, $a = \sqrt{\frac{6\mu^2}{\lambda}}$

The minimum of the potential is at $r = a$. So, define shifted field

$$r(x) = a + h(x),$$

quanta gives Higgs H^0

with all others not shifted

Notice that it is a real $U(1)_Q$ - neutral field $r(x)$ that is shifting $\Rightarrow$ All generators of $SU(2)_L \otimes U(1)_Y$ break except $U(1)_Q$ combo.

The covariant derivative is

$$D_\mu \begin{pmatrix} 0 \\ r \end{pmatrix} = \left(\partial_\mu + \frac{1}{2} i g W_\mu^a \sigma^a + \frac{1}{2} i g' B_\mu \right) \begin{pmatrix} 0 \\ r \end{pmatrix}$$

$$= \left(\begin{array}{c|c} \partial_\mu + \frac{1}{2} i g W_\mu^3 + \frac{1}{2} i g' B_\mu & \frac{1}{2} i g \sqrt{2} W_\mu^+ \\ \hline \frac{1}{2} i g \sqrt{2} W_\mu^- & \partial_\mu - \frac{1}{2} i g W_\mu^3 + \frac{1}{2} i g' B_\mu \end{array} \right) \begin{pmatrix} 0 \\ r \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} i g W_\mu^+ r \\ \partial_\mu r - \frac{1}{2} i g W_\mu^3 r + \frac{1}{2} i g' B_\mu r \end{pmatrix}$$

likewise, $\left[D_\mu \begin{pmatrix} 0 \\ r \end{pmatrix} \right]^\dagger$ can be found.

Find for the Higgs KE term

$$\frac{1}{2} \left[D_\mu \begin{pmatrix} 0 \\ r \end{pmatrix} \right]^\dagger \left[D^\mu \begin{pmatrix} 0 \\ r \end{pmatrix} \right] = \frac{1}{4} g^2 W_\mu^+ W^{\mu -} r^2 + \frac{1}{2} \partial_\mu r \partial^\mu r$$

$$+ \frac{1}{8} r^2 \left(g W_\mu^3 - g' B_\mu \right)^2$$

If $r(x) = a + h(x)$, then about true vacuum

$$\mathcal{L}_{Higgs} = \frac{1}{2}\partial_\mu h \partial^\mu h$$

$$+ \frac{1}{2}(a+h)^2\left[\frac{1}{2}g^2 W_\mu^+ W^{\mu -} + \frac{1}{4}(gW_\mu^3 - g' B_\mu)^2\right]$$

$$- \frac{a^2\lambda}{6} h^2 - \frac{a\lambda}{6} h^3 - \frac{\lambda}{4!} h^4$$

We see that $\boxed{m_h = \sqrt{2}\mu}$, as usual.

Since $W_\mu^- = (W_\mu^+)^*$, see that mass for $W_\mu^\pm$ is

$$\boxed{m_{W^\pm} = \frac{1}{2} a g}$$

For W_μ^3, B_μ, the given term has mass mixing!

$\Rightarrow$ Must diagonalize. This is already done, combo in

$\mathcal{L}_{Higgs}$ is the massive boson

Define:

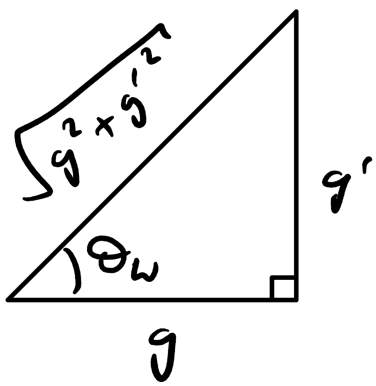
$$\boxed{\begin{aligned} Z_\mu^0 &\equiv \frac{gW_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \\ A_\mu &= \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}} \end{aligned}}$$

So, $\mathcal{L}_{Higgs} \supset \frac{1}{2} a^2 \left(\frac{1}{4} (g W_\mu^3 - g' B_\mu)^2 \right)$

$$= \frac{1}{2} \frac{1}{4} a^2 (g^2 + g'^2) Z_\mu^0 Z^{\mu 0} + 0 \cdot A_\mu A^\mu$$

So, $\boxed{\begin{aligned} m_A &= 0 \\ m_{Z^0} &= \frac{1}{2} a \sqrt{g^2 + g'^2} \end{aligned}}$

Define the weak mixing angle θ_W (also called Weinberg angle)



so that

$$\begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

$$\Rightarrow \boxed{m_{W^\pm} = m_{Z^0} \cos\theta_W}, \quad \boxed{g' = g \tan\theta_W}$$

So, complete Higgs Lagrangian

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}\partial_\mu h \partial^\mu h$$

$$+ \frac{1}{2}(a+h)^2 \left[\frac{1}{2} g^2 W_\mu^+ W^{\mu -} + \frac{1}{4}(g^2+g'^2) Z_\mu^\circ Z^{\circ\mu} \right]$$

$$- \frac{a^2\lambda}{6} h^2 - \frac{a\lambda}{6} h^3 - \frac{\lambda}{4!} h^4$$

$$= \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2} m_h^2 h^2$$

$$+ m_{W^\pm}^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_{Z^\circ}^2 Z_\mu^\circ Z^{\circ\mu}$$

$$+ \frac{1}{4} g^2 h^2 W_\mu^+ W^{-\mu} + \frac{1}{8}(g^2+g'^2) h^2 Z_\mu^\circ Z^{\circ\mu}$$

$$+ \frac{1}{2} a g^2 h W_\mu^+ W^{-\mu} + \frac{1}{4} a (g^2+g'^2) h Z_\mu^\circ Z^{\circ\mu}$$

$$- \frac{a\lambda}{6} h^3 - \frac{\lambda}{4!} h^4$$

With $m_{W^\pm} = \frac{1}{2} a g$, $m_{Z^\circ} = \frac{1}{2} a \sqrt{g^2+g'^2}$

$m_h = \sqrt{2}\mu$, $\mu = \frac{a\lambda}{\sqrt{6}}$

$g' = g \tan\theta_W$, $m_{W^\pm} = m_{Z^\circ} \cos\theta_W$

What about other terms in the EW Lagrange density?

Consider first gauge KE terms,

$$\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu}^{a} W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

transforming the fields,

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix}$$

After considerable algebra, find

$$\begin{aligned} \mathcal{L}_{gauge} = & -\frac{1}{4}\left(\partial_\mu Z_\nu^0 - \partial_\nu Z_\mu^0 - ig\cos\theta_W \left(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+ \right) \right)^2 \\ & -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\mu A_\nu - ie\left(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+ \right) \right)^2 \\ & -\frac{1}{2}\Big| \partial_\nu W_\mu^+ - \partial_\mu W_\nu^+ + ig\cos\theta_W \left(W_\mu^+ Z_\nu^0 - W_\nu^+ Z_\mu^0 \right) \\ & \qquad + ie\left(W_\mu^+ A_\nu - W_\nu^+ A_\mu \right) \Big|^2 \end{aligned}$$

where $\boxed{e \equiv g\sin\theta_W = g'\cos\theta_W}$

↳ electric charge!

Next, consider the fermion KE terms,

$Y = -1$ for L_A

$$D_\mu L_A = \left(\partial_\mu + \frac{1}{2} i g W_\mu^a \sigma^a + \frac{1}{2} i g' B_\mu Y\right)\begin{pmatrix} \nu_{A_L} \\ \ell_{A_L} \end{pmatrix}$$

$$= \begin{pmatrix} \partial_\mu + \frac{1}{2} i g W_\mu^3 - \frac{1}{2} i g' B_\mu & \frac{1}{2} i g \sqrt{2}\, W_\mu^+ \\ \frac{1}{2} i g \sqrt{2}\, W_\mu^- & \partial_\mu - \frac{1}{2} i g W_\mu^3 - \frac{1}{2} i g' B_\mu \end{pmatrix}\begin{pmatrix} \nu_{A_L} \\ \ell_{A_L} \end{pmatrix}$$

$$= \begin{pmatrix} \left(\partial_\mu + \frac{1}{2} i g W_\mu^3 - \frac{1}{2} i g' B_\mu\right)\nu_{A_L} \\ \left(\partial_\mu - \frac{1}{2} i g W_\mu^3 - \frac{1}{2} i g' B_\mu\right)\ell_{A_L} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} i g \sqrt{2}\, W_\mu^+ \ell_{A_L} \\ \frac{1}{2} i g \sqrt{2}\, W_\mu^- \nu_{A_L} \end{pmatrix}$$

So,

$$\bar{L}_A \not{D} L_A = \left(\bar{\nu}_{A_L} \ \ \bar{\ell}_{A_L}\right)\gamma^\mu D_\mu \begin{pmatrix} \nu_{A_L} \\ \ell_{A_L} \end{pmatrix}$$

$$= \bar{\nu}_{A_L}\gamma^\mu\left(\partial_\mu + \frac{1}{2} i g W_\mu^3 - \frac{1}{2} i g' B_\mu\right)\nu_{A_L}$$

$$+ \bar{\ell}_{A_L}\gamma^\mu\left(\partial_\mu - \frac{1}{2} i g W_\mu^3 - \frac{1}{2} i g' B_\mu\right)\ell_{A_L}$$

$$+ \bar{\nu}_{A_L}\gamma^\mu \frac{1}{\sqrt{2}} i g W_\mu^+ \ell_{A_L} + \bar{\ell}_{A_L}\gamma^\mu \frac{1}{\sqrt{2}} i g W_\mu^- \nu_{A_L}$$

Note that $\quad g W_\mu^3 - g' B_\mu = \sqrt{g^2 + g'^2}\, Z_\mu^0$

$$= \frac{g}{\cos\theta_W} Z_\mu^0$$

also,

$$g W_\mu^3 + g' B_\mu = g(\sin\theta_W A_\mu + \cos\theta_W Z_\mu^0) + g'(\cos\theta_W A_\mu - \sin\theta_W Z_\mu^0)$$

$$= (g\sin\theta_W + g'\cos\theta_W) A_\mu + (g\cos\theta_W - g'\sin\theta_W) Z_\mu^0$$

Recall that $g\sin\theta_W = g'\cos\theta_W = e$

also, $g' = g\tan\theta_W$

so, $$g W_\mu^3 + g' B_\mu = 2 e A_\mu + \frac{g}{\cos\theta_W}(\cos^2\theta_W - \sin^2\theta_W) Z_\mu^0$$

$$= 2 e A_\mu + \frac{g}{\cos\theta_W}(1 - 2\sin^2\theta_W) Z_\mu^0$$

Therefore,

$$\bar{L}_A \not{D} L_A = \bar{\nu}_{A_L} \gamma^\mu \left(\partial_\mu + \frac{1}{2} i \frac{g}{\cos\theta_W} Z_\mu^0 \right) \nu_{A_L}$$

$$+ \bar{l}_{A_L} \gamma^\mu \left(\partial_\mu - i e A_\mu - \frac{1}{2} i \frac{g}{\cos\theta_W} (1 - 2\sin^2\theta_W) Z_\mu^0 \right) l_A$$

$$+ \bar{\nu}_{A_L} \gamma^\mu \frac{1}{\sqrt{2}} i g W_\mu^+ l_{A_L} + \bar{l}_{A_L} \gamma^\mu \frac{1}{\sqrt{2}} i g W_\mu^- \nu_{A_L}$$

For the right-handed fields,

-2 for R_A

$$\bar{R}_A \not{D} R_A = \bar{\ell}_{A_R} \gamma^\mu \left(\partial_\mu + \frac{1}{2} i g' B_\mu Y \right) \ell_{A_R}$$

$$= \bar{\ell}_{A_R} \gamma^\mu \left(\partial_\mu - i g' \left(\cos\theta_W A_\mu - \sin\theta_W Z_\mu^0 \right) \right) \ell_{A_R}$$

$$= \bar{\ell}_{A_R} \gamma^\mu \left(\partial_\mu - i e A_\mu + i g' \sin\theta_W Z_\mu^0 \right) \ell_{A_R}$$

$$g' \sin\theta_W = \frac{g \sin^2\theta_W}{\cos\theta_W}$$

So, $\bar{L}_A \not{D} L_A + \bar{R}_A \not{D} R_A$

$$= \bar{\nu}_{A_L} \gamma^\mu \left(\partial_\mu + \frac{1}{2} i \frac{g}{\cos\theta_W} Z_\mu^0 \right) \nu_{A_L}$$

$$+ \bar{\ell}_{A_L} \gamma^\mu \left(\partial_\mu - i e A_\mu - \frac{1}{2} i \frac{g}{\cos\theta_W} (1 - 2\sin^2\theta_W) Z_\mu^0 \right) \ell_A$$

$$+ \bar{\nu}_{A_L} \gamma^\mu \frac{1}{\sqrt{2}} i g W_\mu^+ \ell_{A_L} + \bar{\ell}_{A_L} \gamma^\mu \frac{1}{\sqrt{2}} i g W_\mu^- \nu_{A_L}$$

$$+ \bar{\ell}_{A_R} \gamma^\mu \left(\partial_\mu - i e A_\mu + \frac{i g}{\cos\theta_W} \sin^2\theta_W Z_\mu^0 \right) \ell_{A_R}$$

Separate into EM current, $\sim J^\mu_{em} A_\mu$

weak charged current, $\sim J^\mu_+ W^+_\mu + h.c.$

weak neutral current, $\sim J^\mu_Z Z^0_\mu$

For electromagnetic, find

$$\mathcal{L}_{em} = \bar{l}_{A_L}\gamma^\mu(-ieA_\mu)l_{A_L} + \bar{l}_{A_R}\gamma^\mu(-ieA_\mu)l_{A_R}$$

Recall that $\bar{\psi}_L\gamma^\mu\psi_R = \bar{\psi}_R\gamma^\mu\psi_L = 0$

$$\Rightarrow \mathcal{L}_{em} = \bar{l}_A\gamma^\mu(-ieA_\mu)l_A$$

↳ em current w/ charge $Q=-1$!

$$= -ieJ^\mu_{em}A_\mu$$

↳ $J^\mu_{em} \equiv \bar{l}_A\gamma^\mu l_A$

In general, find $\boxed{\mathcal{L}_{em} = +iQeJ^\mu_{em}A_\mu}$

↳ $Q=-1$ for l_A

For weak charged interactions,

$$\mathcal{L}_{cc} = \bar{\nu}_{A_L}\gamma^\mu\frac{1}{\sqrt{2}}igW_\mu^+ l_{A_L} + \bar{l}_{A_L}\gamma^\mu\frac{1}{\sqrt{2}}igW_\mu^-\nu_{A_L}$$

Now, $l_{A_L} = P_L l_A$, $\bar{\nu}_{A_L} = \bar{\nu}_A P_R$

But, $P_R\gamma^\mu P_L = \gamma^\mu P_L^2 = \gamma^\mu P_L$

$$\Rightarrow \mathcal{L}_{cc} = \frac{ig}{\sqrt{2}}\bar{\nu}_A\gamma^\mu P_L l_A W_\mu^+ + \frac{ig}{\sqrt{2}}\bar{l}_A\gamma^\mu P_L\nu_A W_\mu^-$$

Since $P_L = \frac{1}{2}(1-\gamma_5)$, then

$$\mathcal{L}_{cc} = \frac{ig}{\sqrt{2}} J_-^\mu W_\mu^+ + \frac{ig}{\sqrt{2}} J_+^\mu W_\mu^-$$

where,

$$\boxed{\begin{aligned} J_-^\mu &= \frac{1}{2}\bar{\nu}_A \gamma^\mu (1-\gamma_5) \ell_A \\ J_+^\mu &= \frac{1}{2}\bar{\ell}_A \gamma^\mu (1-\gamma_5) \nu_A \end{aligned}}$$

↳ V-A interaction!

For weak neutral currents,

$$\mathcal{L}_{nc} = \bar{\nu}_{A_L}\gamma^\mu\left(\frac{1}{2} i \frac{g}{\cos\theta_W} Z_\mu^0\right)\nu_{A_L}$$
$$+ \bar{\ell}_{A_L}\gamma^\mu\left(-\frac{1}{2} i \frac{g}{\cos\theta_W}(1-2\sin^2\theta_W) Z_\mu^0\right)\ell_A$$
$$+ \bar{\ell}_{A_R}\gamma^\mu\left(\frac{ig}{\cos\theta_W}\sin^2\theta_W\, Z_\mu^0\right)\ell_{A_R}$$

For neutrinos,

$$\bar{\nu}_{A_L}\gamma^\mu \frac{1}{2}\frac{ig}{\cos\theta_W} Z_\mu^0 \nu_{A_L} = \frac{ig}{\cos\theta_W}\bar{\nu}_A \gamma^\mu\left(\frac{1}{4}-\frac{1}{4}\gamma_5\right)\nu_A Z_\mu^0$$

For leptons,

$$\frac{ig}{\cos\theta_W}\,\bar{\ell}\,\gamma^\mu\,\frac{1}{2}(1-\gamma_5)\left[-\frac{1}{2}+\sin^2\theta_W\right]\ell\, Z_\mu^0$$

$$+\,\frac{ig}{\cos\theta_W}\,\bar{\ell}\,\gamma^\mu\,\frac{1}{2}(1+\gamma_5)\,\sin^2\theta_W\,\ell\, Z_\mu^0$$

$$=\frac{ig}{\cos\theta_W}\,\bar{\ell}\,\gamma^\mu\left(-\frac{1}{4}+\sin^2\theta_W+\frac{1}{4}\gamma_5\right)\ell\, Z_\mu^0$$

In general,

$$\mathcal{L}_{nc} \supset \frac{ig}{\cos\theta_W} J_Z^\mu Z_\mu^0$$

with $\boxed{J_Z^\mu = \bar{f}\gamma^\mu(v_f - a_f\gamma_5)f}$

V−A interaction

f	v_f	a_f
ν	$\frac{1}{4}$	$\frac{1}{4}$
ℓ	$-\frac{1}{4}+\sin^2\theta_W$	$-\frac{1}{4}$

Note that the KE terms are as usual,

$$\bar{\ell}_A \not{\partial} \ell_A , \quad \bar{\nu}_A \not{\partial} \nu_A$$

where we note that since ν_A are only left-handed,

$$\nu_{A_L} = \nu_A .$$

Finally, the Yukawa terms change after SSB,

$$\mathcal{L}_{Yukawa} = G_A^L [\bar{R}_A (\phi^\dagger L_A) + (\bar{L}_A \phi) R_A]$$

$$\underset{\substack{\uparrow \\ \text{unitary gauge}}}{\phi =} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ r \end{pmatrix} \underset{\substack{\uparrow \\ \text{SSB}}}{\rightarrow} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ a+h \end{pmatrix}$$

So, $$\phi^\dagger L_A = \frac{1}{\sqrt{2}} (0, \underset{\substack{\uparrow \\ \text{Higgs is real}}}{a+h}) \begin{pmatrix} \nu_{A_L} \\ \ell_{A_L} \end{pmatrix} = \frac{1}{\sqrt{2}} (a+h) \ell_{A_L}$$

Similarly,

$$\bar{L}_A \phi = \frac{1}{\sqrt{2}} (\bar{\nu}_{A_L}, \bar{\ell}_{A_L}) \begin{pmatrix} 0 \\ a+h \end{pmatrix} = \frac{1}{\sqrt{2}} (a+h) \bar{\ell}_{A_L}$$

So,

$$\begin{aligned} \mathcal{L}_{Yukawa} &= G_A^L [\bar{R}_A (\phi^\dagger L_A) + (\bar{L}_A \phi) R_A] \\ &= \frac{1}{\sqrt{2}} G_A^L (a+h) [\bar{\ell}_{A_R} \ell_{A_L} + \bar{\ell}_{A_L} \ell_{A_R}] \\ &= \frac{1}{\sqrt{2}} G_A^L (a+h) \bar{\ell}_A \ell_A \end{aligned}$$

Recall $\bar{\psi}_L \psi_L = \bar{\psi}_R \psi_R = 0$

Therefore, get mass term for lepton & higgs interaction,

$$\mathcal{L}_{Yukawa} = m_{\ell_A} \bar{\ell}_A \ell_A + \frac{m_{\ell_A}}{a} h \bar{\ell}_A \ell_A$$

where,
$$\boxed{m_{\ell_A} \equiv \frac{1}{\sqrt{2}} G_A^L a}$$

$$\hookrightarrow = \frac{1}{2} \frac{m_{\ell_A}}{m_W} g \, h \bar{\ell}_A \ell$$

w/ $m_W = \frac{1}{2} a g$

Note: could include generation mixing,

$$\sim G_{AB}^L \bar{R}_A \phi L_B + h.c.$$

But, since there are no right-handed neutrinos, we can diagonalize it. When we include quarks, this will not be the case for them.

EW Parameters

There are 7 EW parameters,

$$g, g', \mu^2, \lambda, G_e^L, G_\mu^L, G_\tau^L.$$

SSB introduces many relations between these and other physical parameters.

The 3 Yukawa couplings are fixed by the lepton masses,

$$\boxed{\begin{aligned} m_e &\rightarrow G_e^L \\ m_\mu &\rightarrow G_\mu^L \\ m_\tau &\rightarrow G_\tau^L \end{aligned}}$$

The remaining couplings, g, g', λ, μ, need to be fixed from other observables. The weak mixing angle, for example, has been measured from many processes involving ν_f, e_f in different ways, $\boxed{\sin^2\theta_W \simeq 0.23}$

From measuring $e \simeq 0.303$

$$= g \sin\theta_W \Rightarrow \boxed{g = \frac{e}{\sin\theta_W} \simeq 0.63}$$

Furthermore, $e = g'\cos\theta_W \Rightarrow \boxed{g' = \frac{e}{\sqrt{1-\sin^2\theta_W}} \simeq 0.35}$

From muon decay, can measure <u>Fermi Constant</u>

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} \simeq \frac{1}{\sqrt{2}}\left(1.166\times 10^{-5}\ \mathrm{GeV}^{-2}\right)}$$

Since $m_W = \frac{1}{2} g a$, the Higgs vev is

$$a = 2 \frac{m_W}{g} = 2\sqrt{\frac{\sqrt{2}}{8 G_F}} = (\sqrt{2}\, G_F)^{-1/2}$$

So,

$$\boxed{a = v \simeq 246 \text{ GeV}}$$

for 2012 discovery!

The Higgs mass is $\boxed{m_H \simeq 125 \text{ GeV}}$,

So, $m_h = \sqrt{2}\,\mu \Rightarrow \boxed{\mu = \frac{1}{\sqrt{2}} m_h \simeq 88 \text{ GeV}}$

Since Higgs vev is

$$a = \sqrt{\frac{6\mu^2}{\lambda}} \Rightarrow \boxed{\lambda = \frac{6\mu^2}{a^2} \simeq 0.77}$$

So, the 7 independent parameters can be determined by various measurements. All other quantities can be determined from these, e.g.,

$$\boxed{\begin{aligned} m_{W^\pm} &\simeq 80.42(4) \text{ GeV} \\ m_Z &\simeq 91.188(2) \text{ GeV} \end{aligned}}$$