

Hadrons

Beginning in the 1950's, there were 100's of particles being discovered. It was observed that these new particles did not interact like electrons or photons, and had features similar to the proton and neutron. These particles were called hadrons. Attempts to understand the "Hadron Zoo" led to the applications of symmetry to understand their fundamental interactions.

Hadrons come in two types: Mesons and Baryons

To distinguish these two, we introduce the quantum number Baryon number B_n

We assign $B_n = \begin{cases} +1 & \text{for Baryons} \\ -1 & \text{for Anti-Baryons} \end{cases}$

Mesons have $B_n = 0$.

Baryon number is always conserved!

$$\Delta B_n = 0$$

In the SM, Baryon number is represented by a global $U(1)$ symmetry of QCD.

The fact that $\Delta B_n = 0$ in reactions means that reactions like $p \rightarrow e^+ + \pi^0$ are not observed!

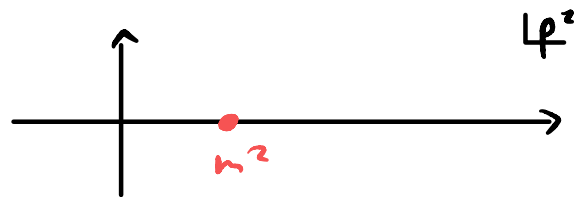
In the universe we observe more matter than antimatter. Baryogenesis is the generation of this asymmetry. One of the conditions for such a process is $\Delta B_n \neq 0$ (along with C & CP violation).

Resonances

How do we detect these short-lived hadrons? Strangely decaying hadrons are resonances of scattering processes.

Consider a propagator for a stable particle,

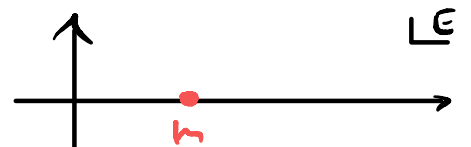
$$i\Delta = \frac{i}{p^2 - m^2}$$



The associated wave function is $\langle 0 | \hat{\varphi}(x) | p \rangle = e^{-ip \cdot x}$

For a state at rest, $p = (E, \vec{0})$, and for energies

near the pole, $i\Delta \sim \frac{i}{2m(E - m)}$



In NRQM, the wave function gives probability amplitude,

$$\psi^* \psi \sim e^{i\omega t} e^{-i\omega t} = 1$$

What if a particle decays?

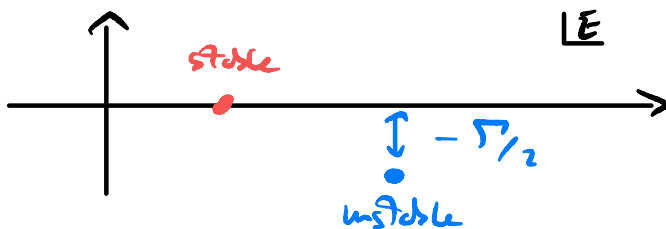
Exped $\psi^* \psi \sim e^{-t/\tau}$

↑ Lifetime of the particle

This effectively shifts the pole into the complex energy plane

$$i\Delta \sim \frac{i}{2m(E - m + i\Gamma/2)}$$

where Γ is the decay width, $\Gamma = 1/\tau$



wave function, $\psi = e^{-iE_{pole}t} = e^{-i(m - i\Gamma/2)t}$

$$\begin{aligned} \Rightarrow \psi^* \psi &= e^{i(m + i\Gamma/2)t} e^{-i(m - i\Gamma/2)t} \\ &= e^{-\Gamma/2 t} e^{-\Gamma/2 t} \\ &= e^{-\Gamma t} = e^{-t/\tau} \quad \checkmark \end{aligned}$$

⇒ unstable hadrons are pole singularities of Green's functions or scattering amplitudes in the complex energy plane

For a narrow, isolated resonance, the Breit-Wigner amplitude phenomenologically parameterizes the amplitude

$$iM_{BW} \sim \frac{1}{s - m^2 + i m \Gamma}$$

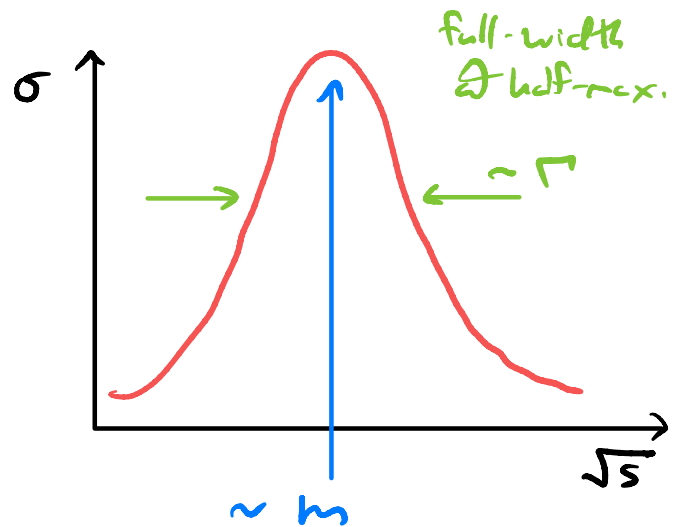
so that

$$\sigma_{BW} = \frac{A}{(s - m^2)^2 + m^2 \Gamma^2}$$

Note: $s = m^2 - i m \Gamma$

$$\Rightarrow E = m \sqrt{1 - i \Gamma / m}$$

$$\approx m - i \Gamma / 2 \quad \leftarrow \text{assumes } m \ll \Gamma$$



Isospin

There are other useful "symmetries" to unravel hadron physics.

Let's consider first the proton and the neutron.

It is observed that $m_p \approx m_n$

$$\left. \begin{array}{l} m_p \approx 938.3 \text{ MeV} \\ m_n \approx 939.6 \text{ MeV} \end{array} \right\} \Rightarrow \frac{\Delta m}{m} \approx 0.14 \%$$

This suggests that p, n as having an "approximate" symmetry under strong interaction properties

\Rightarrow p, n are 2 aspects of "single" particle N, "nucleon"

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \text{a doublet}$$

Similarly: $\pi^+ \pi^0 \pi^- \Rightarrow \frac{\Delta m}{m} \sim 3.3\%$

$\Delta^{++} \Delta^+ \Delta^0 \Delta^- \Rightarrow \frac{\Delta m}{m} \sim 1\%$

etc.

It is observed that the strong interaction cannot distinguish components of these "grouped" particles, e.g.:

$$\begin{aligned} \pi^+ p &\sim \pi^+ n \\ &\sim \pi^0 p \sim \pi^0 n \\ &\sim \pi^- p \sim \pi^- n \end{aligned}$$

To distinguish the states, introduce the idea of isospin $\vec{I} = (I_1, I_2, I_3)$, generated

by $su(2)$ algebra $[I_j, I_k] = i\epsilon_{jkl} I_l$.

States labeled by eigenvalues of $\vec{I}^2 (i(i+1))$

and $I_3 (i_3)$

Explicitly, $N \quad i = \frac{1}{2} \quad i_3 = \begin{cases} +\frac{1}{2} & p \\ -\frac{1}{2} & n \end{cases}$

$\pi \quad i = 1 \quad i_3 = \begin{cases} +1 & \pi^+ \\ 0 & \pi^0 \\ -1 & \pi^- \end{cases}$

Strong isospin only applies to hadrons. For
non-strange hadrons,

$$Q = I_3 + \frac{1}{2} B_n$$

electric charge \leftarrow

\rightarrow Baryon number

Can learn a lot about reactions just by isospin
considerations. If SM Hamiltonian is decomposed
as $H_{SM} = H_s + H_{EM} + H_w$

then $[H_s, \vec{I}] = 0$ since we defined isospin to
be conserved in strong interactions. Now, consider
the Nucleon N , $N = (p, n)$. Isospin is broken
explicitly by EM interactions $\Rightarrow [H_{EM}, \vec{I}] \neq 0$
since one state is charged. But, $Q = I_3 + \frac{1}{2} B$
and Q & B are good quantum numbers $\Rightarrow I_3$ is
a good quantum number $\Rightarrow \underline{[H_{EM}, I_3] = 0}$!

But, it is observed that $[H_w, \vec{I}] \neq 0$ & $[H_w, I_3] \neq 0$
 \Rightarrow isospin is completely broken by weak interactions

Example

Consider $\Lambda^0 \rightarrow p \pi^-$ decay (BR $\sim 64\%$)

Estimate lifetime of Λ^0

Consider isospin numbers

$$\Lambda^0 \rightarrow p \pi^-$$
$$i: 0 \rightarrow \frac{1}{2} \quad 1 \quad \leftarrow \Delta I \neq 0 \text{ since } \underline{2} \times \underline{3} \neq \underline{1}$$
$$i_3: 0 \rightarrow \frac{1}{2} \quad -1 \quad \leftarrow \Delta I_3 \neq 0$$

Therefore, since \vec{I} & I_3 not conserved, this must be a weak decay!

$$\Rightarrow \tau_{\Lambda^0} \sim 10^{-9} \text{ sec} \quad (\tau_{\pi^0} = 2.6 \times 10^{-10} \text{ s})$$

Example

Consider deuteron production, $NN \rightarrow \pi d$

where deuteron is a bound state of p and n.

Given that deuteron has $i_d = 0$, estimate

the ratio of cross-sections σ_a / σ_b where

$$a: pp \rightarrow \pi^+ d$$

$$b: np \rightarrow \pi^0 d$$

Recall that

$$\sigma \sim |\langle T | T | i \rangle|^2 \times (\text{kinematic factors})$$

To good approx, only difference between σ_a & σ_b comes from the amplitude. Moreover, the only difference is in $|i\rangle$ and $|f\rangle$, which is contained in Clebsch-Gordan coefficients

$$\text{Here, } N \quad N \rightarrow \pi \quad d$$

$$i: \quad \frac{1}{2} \quad \frac{1}{2} \rightarrow 1 \quad 0$$

restricted to $i=1$ only \Leftrightarrow $\left\{ \begin{array}{l} \text{This means } 2 \times 1 = 1 \\ \text{only for } su(2)_I \end{array} \right.$

So, for $i=1$ for this process, find

a:

$$\rho \quad \rho \quad | \frac{1}{2} \frac{1}{2} \rangle \times | \frac{1}{2} \frac{1}{2} \rangle = | 1 1 \rangle$$

$$\pi^+ \quad d \quad | 1 1 \rangle \times | 0 0 \rangle = | 1 1 \rangle$$

b:

$$\pi \quad \rho \quad | \frac{1}{2} -\frac{1}{2} \rangle \times | \frac{1}{2} \frac{1}{2} \rangle = \frac{1}{\sqrt{2}} (| 1 0 \rangle - | 0 0 \rangle)$$

$$\pi^- \quad d \quad | 1 0 \rangle \times | 0 0 \rangle = | 1 0 \rangle$$

So, $\frac{\sigma_a}{\sigma_b} = \left(\frac{1}{\sqrt{2}} \right)^2 = 2$, which is experimentally observed.

Isospin is part of a larger concept called Flavor symmetry. Another quantum number, Strangeness S, was assigned to hadrons produced by strong interactions, yet decayed via weak interactions.

Examples of strange hadrons include:

$$K^0, K^+ - S = +1$$

$$\bar{K}^0, K^- - S = -1$$

$$\Lambda^0 - S = -1 \Rightarrow \Lambda^0 \rightarrow \begin{cases} p + \pi^- & \sim 64\% \\ n + \pi^0 & \sim 36\% \end{cases}$$

$\Delta S \neq 0 \Rightarrow$ weak decay
 \Rightarrow long lifetime

$$\tau_{\Lambda^0} \sim 2.6 \times 10^{-10} \text{ s}$$

Common to combine S & B_n into Hypercharge Y

$$Y = B_n + S$$

There are other flavor quantum numbers, charmness C and bottomness B. We will see these are associated w/ the quark content. Won't discuss now on these for time.

$$Q = I_3 + \frac{1}{2} Y, \quad Y = B_n + S + C + B$$

G-parity

There is another discrete quantum number useful for strong interactions, G-parity. This is an extension of C-parity valid for states w/ $Q \neq 0$ ($B=S=0$ still) and $I \neq 0$.

Df: $G = C \exp(i\pi I_2)$

↑
charge conjugation

↑ rotation by π in isospin space around "2"-axis.

Consider some state $|i_3\rangle$

$$G|i_3\rangle = C \exp(i\pi I_2)|i_3\rangle$$

$$\propto C|-i_3\rangle$$

$$\propto |i_3\rangle \quad \text{An eigenstate!}$$

Example: The $\pi = (\pi^+, \pi^0, \pi^-)$ multiplet.

Since $C|\pi^0\rangle = +|\pi^0\rangle$

↑ definition

$$R|i_3\rangle = (-1)^{i_3} |i_3\rangle$$

and

$$R|\pi^0\rangle = R|10\rangle = (-1)^1 |10\rangle = -|10\rangle = -|\pi^0\rangle$$

" $\exp(i\pi I_2)$

$$\Rightarrow G|\pi^0\rangle = -|\pi^0\rangle$$

So, assign $G|\pi^\pm\rangle = -|\pi^\pm\rangle$

Therefore, we have for the π -multiplet

$$G|\pi\rangle = -|\pi\rangle$$

$$\Rightarrow G|n\pi\rangle = (-1)^n |n\pi\rangle$$

\uparrow
 $n = \text{number of pions}$

States w/ $B_n \neq 0$ or $S \neq 0$ cannot be eigenstates of G -parity. This doesn't really add much new as it really is a consequence of isospin, but it gives a shortcut in analyzing reactions.

e.g., what type of decay is $\eta^0 \rightarrow 3\pi$?

$$\begin{array}{l} \eta^0 \rightarrow \pi^+ \pi^0 \pi^- \\ \text{is } 0 \rightarrow +1 \quad 0 \quad -1 \quad \Delta I_3 = 0 \end{array}$$

strang? No!

\Rightarrow Not strong decay!

$$G|\eta^0\rangle = +|\eta^0\rangle, \quad G|3\pi\rangle = (-1)^3 |3\pi\rangle = -|3\pi\rangle$$

\Rightarrow G -parity not conserved! It is electromagnetic

To see that it is not strong w/o G -parity requires analysis of the states angular distribution under generalized Bose statistics.

Flavor SU(3)

We discussed the approximate (or broken) Isospin and Strangeness symmetries, both good for strong interactions. Suggests enlarging $su(2)_I$ to bigger group.

Particles in $su(2)_I$ multiplets are "easy" to find (masses are similar as I-sym only slightly broken). Appropriate choice for multiplets with S harder.

Accepted solution is $SU(3)_F$ (Flavor SU(3)), known as the "Eightfold Way", proposed by Gell-Mann and Ne'eman in 1961.

Note: Do not confuse with $SU(3)_C$ - color

Essential content of $SU(3)_F$

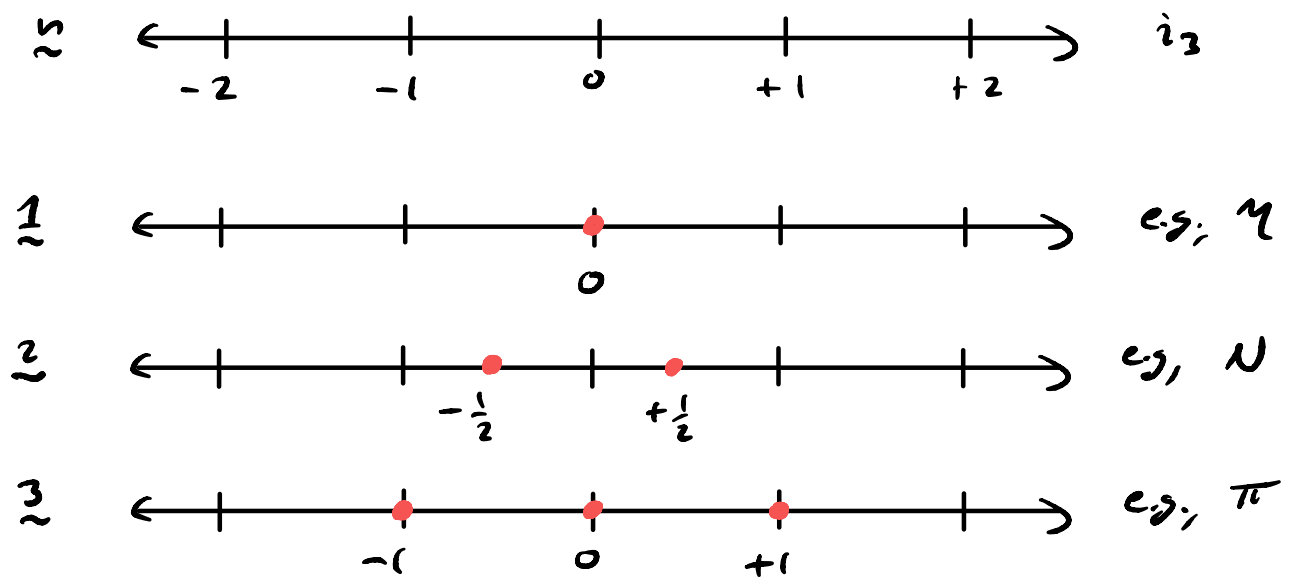
- (1) All baryons fit into $\underline{1}$, $\underline{8}$, $\underline{10}$ of $SU(3)$
- (2) All mesons fit into $\underline{1}$, $\underline{8}$ of $SU(3)$

Quantum numbers of $SU(3)_F$: I, I_3, S or Y, \dots

N.B. There are two Casimirs of $SU(3)$. Here we will focus just on these and the representation.

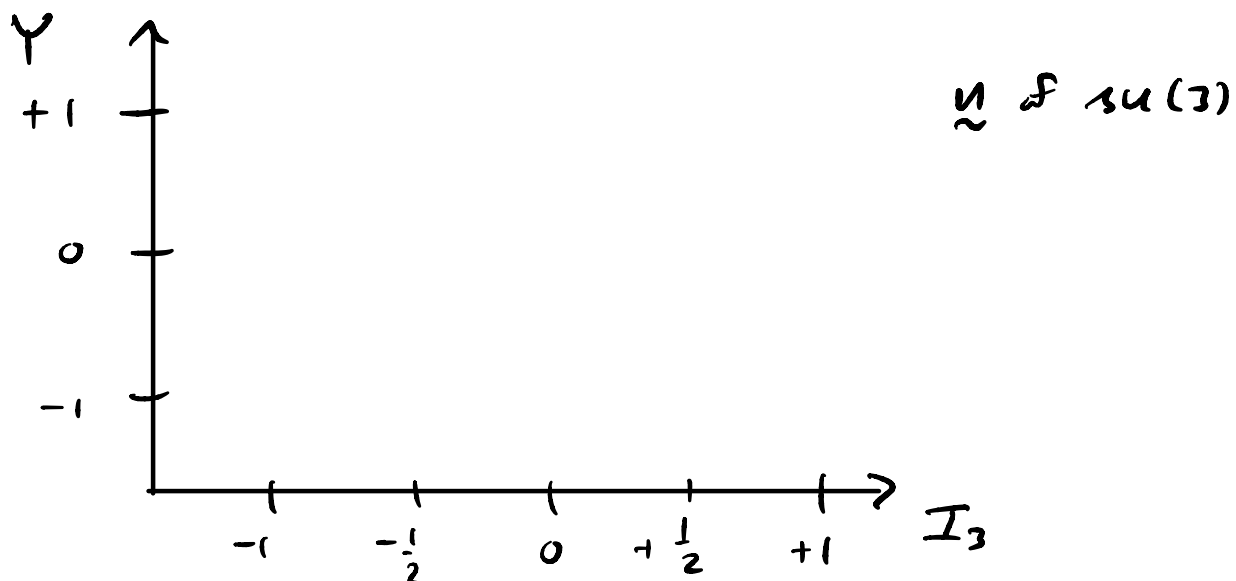
To organize \mathbb{D} 's, can represent a particular rep by a "weight diagram"

For example, $su(2) \Rightarrow$ for given λ , only need one number, i_3 , to classify states

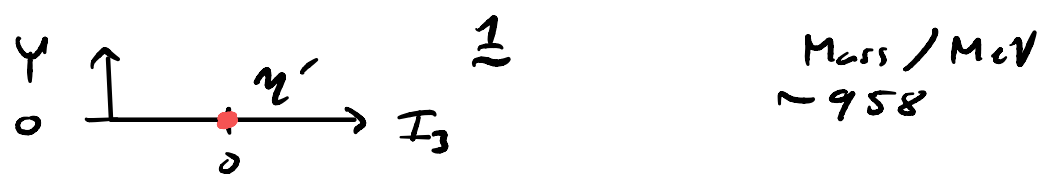


For a given rep in $so(3)$, need two numbers.

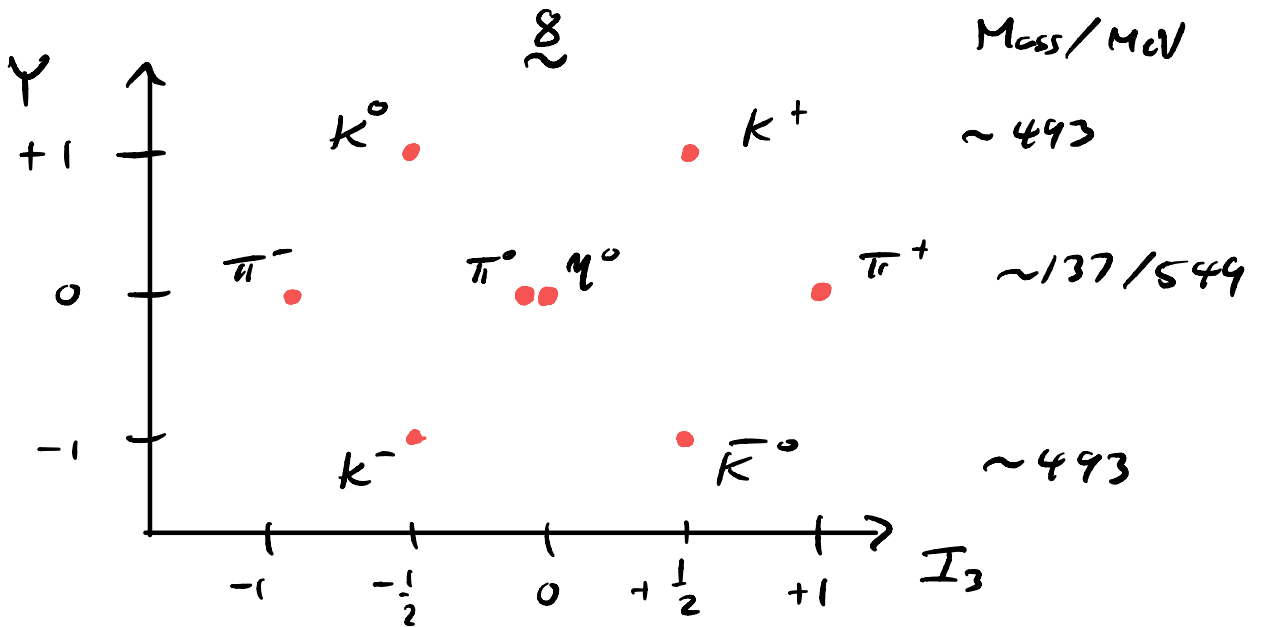
For $su(3)_F$, \mathbb{D} 's take I_3 and Y for a given rep.



Example: The $J^{PC} = 0^{-+}$ meson singlet



Example: The $J^{PC} = 0^{-+}$ meson octet

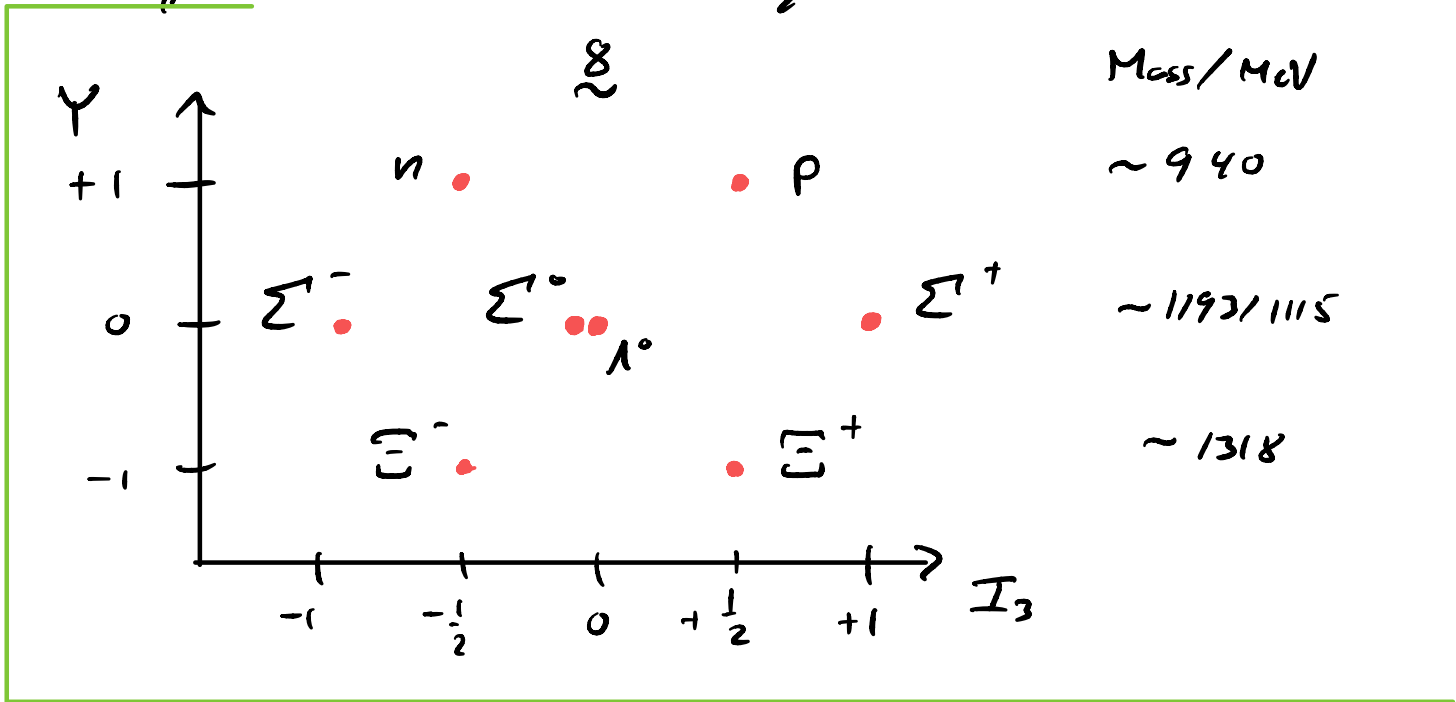


Notice that we have larger mass differences \Rightarrow $SU(3)_F$ is "more broken" than $SU(2)_I$. Also, $SU(2)_I$ multiplets are contained in $SU(3)_F$ ones (proper subgroup).

Notice that the "Gell-Mann / Nishijima" relation holds,

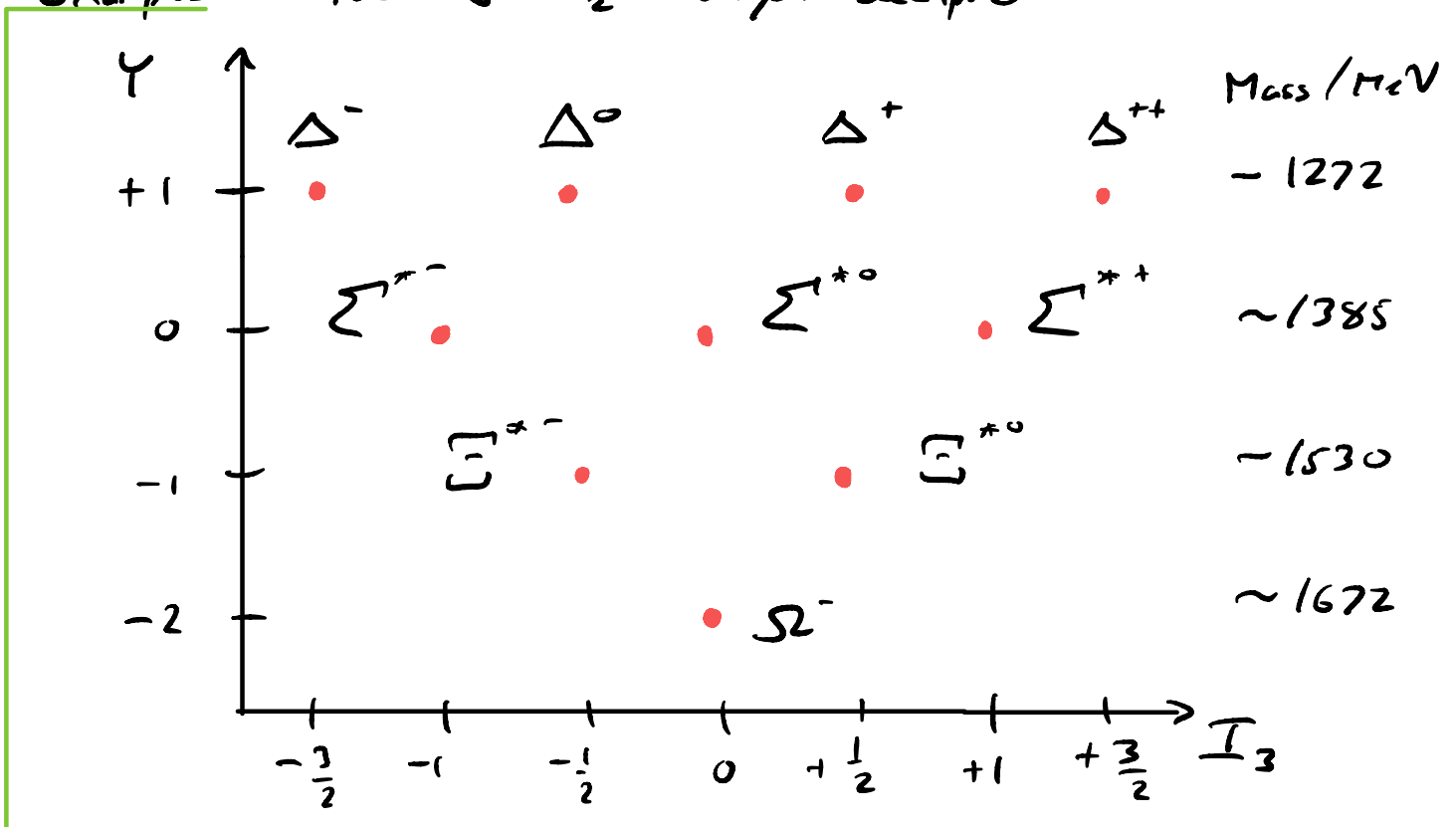
$$Q = I_3 = \frac{1}{2} Y \quad \text{with} \quad Y = B_n + S \quad (\text{check!})$$

Example: The $J^P = \frac{1}{2}^+$ baryon octet

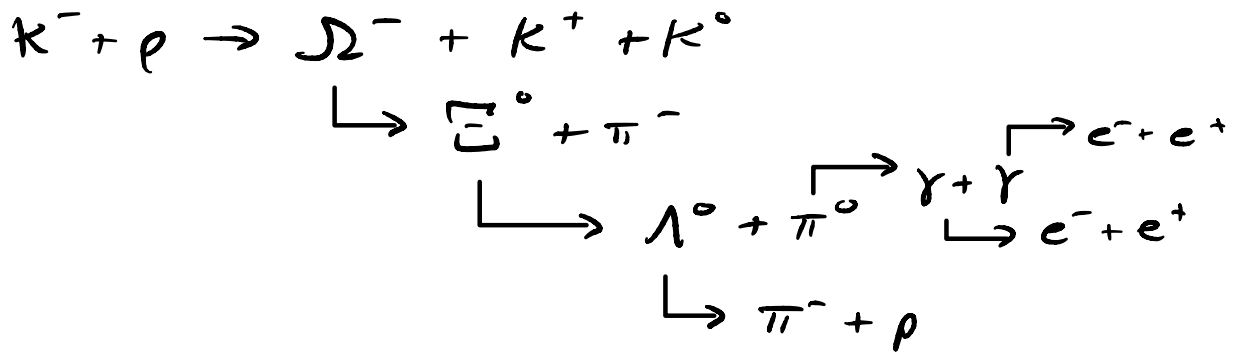


Since $B_n \neq 0$, baryons and antibaryons live in different multiplets. (Exercise - construct antibaryon octet)

Example: The $J^P = \frac{3}{2}^+$ baryon decuplet



The prediction of the Ω^- using $SU(3)_F$ in 1962 and subsequent discovery in 1964 led to the acceptance of the "Eightfold way".



There are many properties of hadrons we can learn simply by examining their symmetry, e.g., magnetic moments of hadrons, mass relations, selection rules of reactions, etc.

The symmetry structures suggest that these hadrons are composite particles of some more fundamental constituents...

The Quark Model

Hypothesis: Hadrons are formed from constituent particles

In 1964, Gell-Mann / Zweig postulated a triplet of fundamental particles with $B_n = 1/3$ to form all hadrons, including p, n, Λ , ...

This is known as the Quark Model

Basic idea,

- Baryons are bound states of qqq ($B_n = 1$)
- Antibaryons are bound states of $\bar{q}\bar{q}\bar{q}$ ($B_n = -1$)
- Mesons are bound states of $q\bar{q}$ ($B_n = 0$)

LJ's focus on 3-flavour hadrons.

So, $q = \{u, d, s\}$ flavours of quarks

\uparrow \uparrow \uparrow
up down strange

Since baryons are fermions, constituent quark (q) is also

fermion, w/ $B_n = \frac{1}{3}$ and $J^P = \frac{1}{2}^+$

N.B. $J = \frac{1}{2}$ is experimentally verified, $P = +$ is a choice.

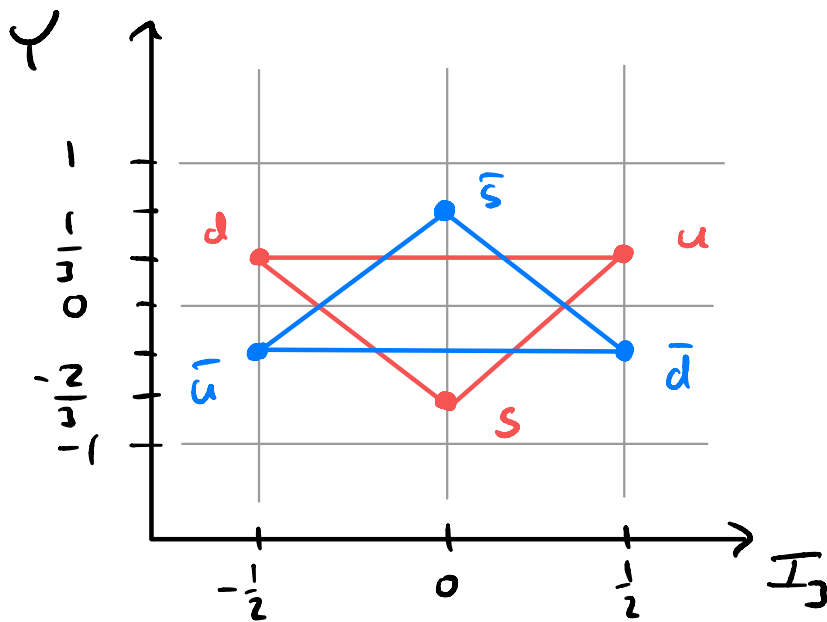
Quark quantum numbers

	I	I_3	$Y = B_n + S$	S	Q	B_n	J^P
u	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}^+$
d	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}^+$
s	0	0	$-\frac{2}{3}$	-1	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}^+$

↑ Notice!

The u, d, s quarks form an $SU(3)_F$ triplet $\underline{3}$

$\Rightarrow \bar{u}, \bar{d}, \bar{s}$ form $\underline{3}^*$ of $SU(3)_F$.



See that quark flavor must be preserved by strong and EM interactions, but can be broken by weak interactions

isospin is still relatively good, $m_u \approx m_d$,

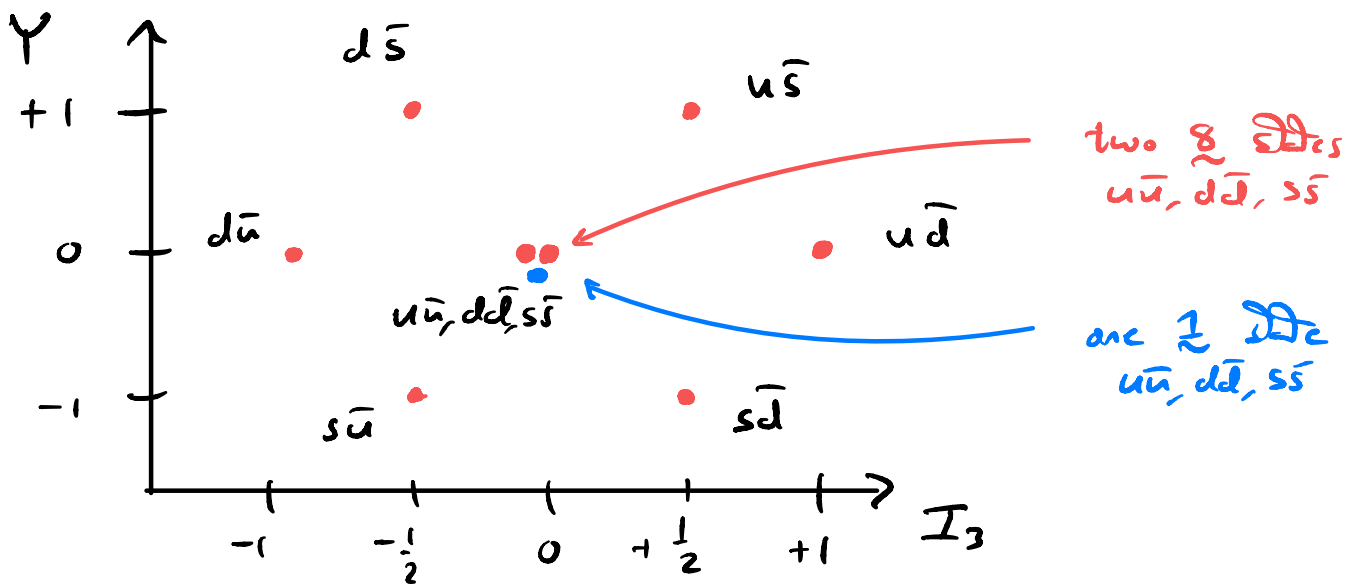
but Y breaking $\Rightarrow m_s$ is different.

Mesons in the Quark Model

A meson is a bound state of $q\bar{q}$

$$\Rightarrow \underline{3} \times \underline{3}^* = \underline{1} + \underline{8}$$

This explains the meson structure in the Eightfold way!

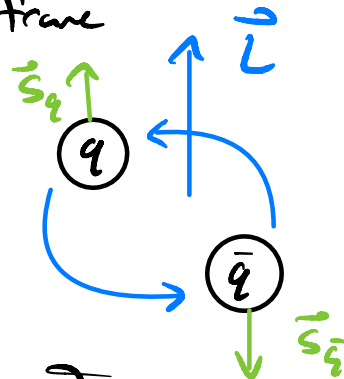


For non-flavour quantum numbers, consider two quarks in centre-of-momentum frame

Since $s_q = s_{\bar{q}} = \frac{1}{2}$

$$\Rightarrow \underline{2} \times \underline{2} = \underline{1} + \underline{3}$$

$$\Rightarrow S = 0 \text{ or } 1$$



$L = 0, 1, 2, \dots$ orbital angular momentum

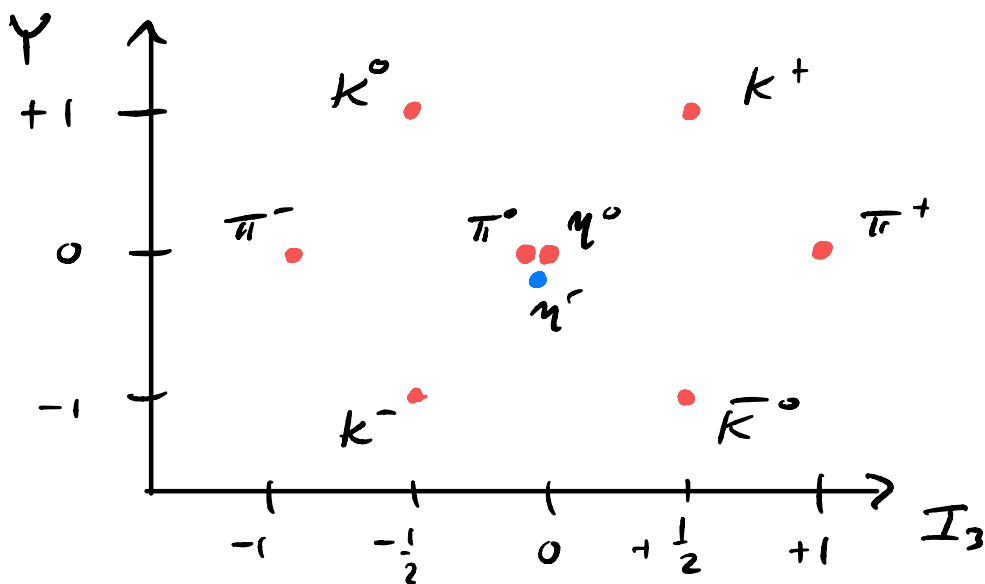
and $\vec{J} = \vec{L} + \vec{S} \Rightarrow |L - S| \leq J \leq L + S$

Can also show that parity $P = (-1)^{L+1}$
 and C-parity $C = (-1)^{L+S}$ (and G-parity $G = (-1)^{I+L+S}$)

For example, the pseudo scalar meson $J^{PC} = 0^{-+}$

$q\bar{q}$ in lowest state $\Rightarrow L=0, S=0$

so, $P = -1, C = +1 \Rightarrow J^{PC}(^{2S+1}L_J) = 0^{-+}(^1S_0)$



$$G_{\pi} = (-1)^{1+0+0} = -1 \quad \checkmark$$

$$G_{\eta} = (-1)^{0+0+0} = +1 \quad \checkmark$$

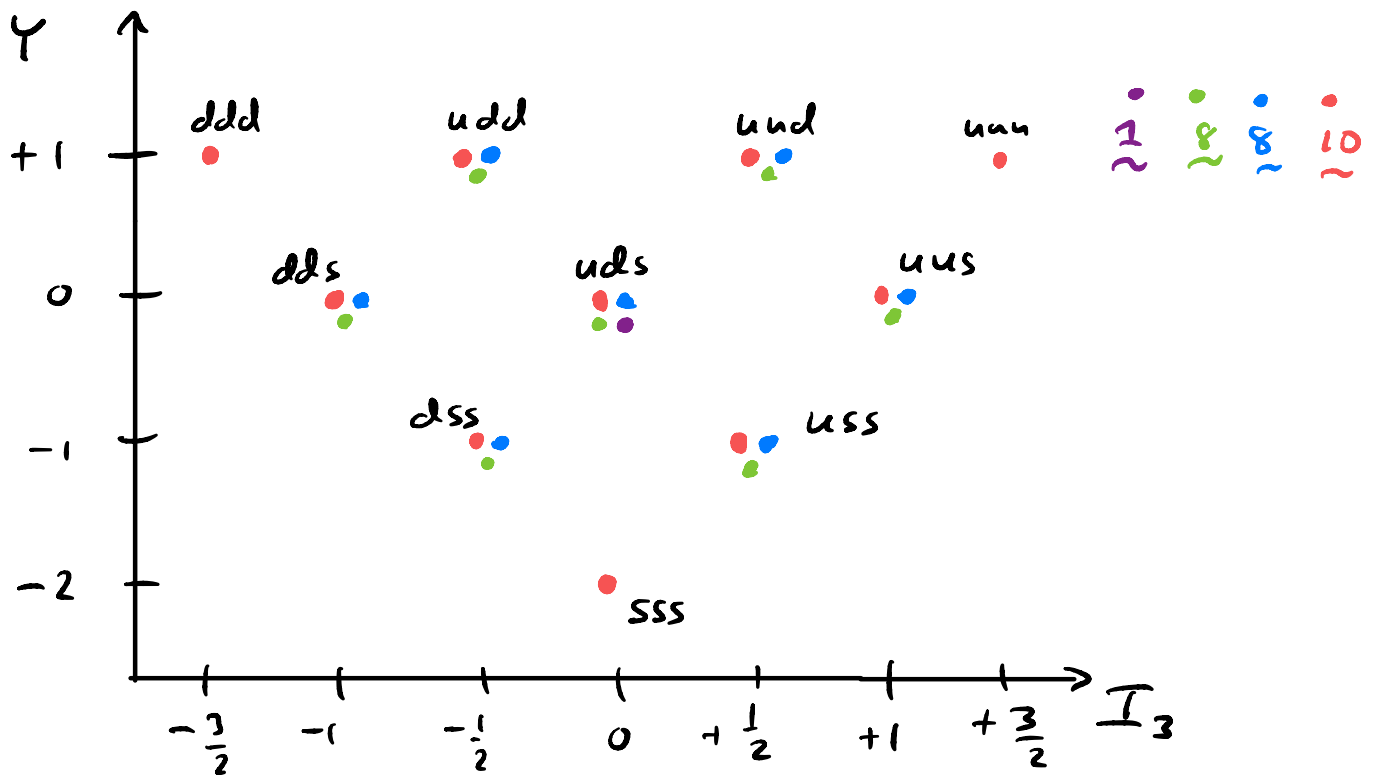
Can add dynamics to quark models to predict decay rates, etc. by assuming some potential in Schrödinger equation. Can get some qualitative understanding, but many issues/failures, e.g., no coupling to multihadron states observed in reactions.

Baryons in the Quark Model

A baryon is a bound state of 3 quarks

$$\Rightarrow \underline{3} \times \underline{3} \times \underline{3} = \underline{1} + \underline{8} + \underline{8} + \underline{10}$$

Again, see Eightfold way arises from constituent quarks!



Can construct quark model wave functions, find some qualitative agreement, e.g., baryon magnetic moments, and can extract constituent quark masses and predict new states.

For example, consider groundstate $J^P = \frac{3}{2}^+$ baryons,

$$\begin{array}{l} \Delta^{++} - I = \frac{3}{2}, m_{\Delta} = 1230 \text{ MeV}, uuu \\ \Delta^- - I = \frac{3}{2}, m_{\Delta^-} = 1230 \text{ MeV}, ddd \\ \Sigma^- - I = 0, m_{\Sigma} \approx 1670 \text{ MeV}, sss \end{array} \left. \vphantom{\begin{array}{l} \Delta^{++} \\ \Delta^- \\ \Sigma^- \end{array}} \right\} \Rightarrow m_u \approx m_d$$

Can deduce that $m_{\Delta} \approx 3m_u \approx 3m_d$

and $m_{\Sigma} \approx 3m_s$

isospin "symmetry"

So, find $m_u \approx m_d = 410 \text{ MeV}$

$m_s \approx 557 \text{ MeV}$

$SU(3)_F$ more broken!

BSB , $m_p = 940 \text{ MeV} \neq 2m_u + m_d$

\Rightarrow seems to indicate dynamics play a bigger role in the binding

Moreover, $m_{\pi} = 140 \text{ MeV} \neq m_u + m_d \approx 820 \text{ MeV}$!

\Rightarrow seems to be a large discrepancy w/ pseudo scalars

In QCD, most of a hadron's energy comes from interactions of quarks (a few MeV) with gluons!

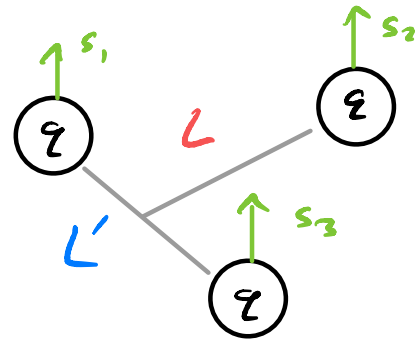
And the pseudo scalars play a role in dynamical chiral symmetry breaking!

Color

There is an interesting puzzle if we look at the Δ^{++} ,
 It is the lowest $\frac{3}{2}^+$ state, doubly charged (peculiar?)
 with quark content uuu . Its spin structure is

$$\begin{aligned} \square_{\underline{2}} \times \square_{\underline{2}} \times \square_{\underline{2}} &= \square_{\underline{2}} \times \left(\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \right) \\ &= \square_{\underline{2}} + \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array} \\ &\quad \underbrace{\hspace{10em}}_{s = \frac{1}{2}} \quad \underbrace{\hspace{10em}}_{s = \frac{3}{2}} \end{aligned}$$

But, since it is the lowest state, expect
 $L = L' = 0$ orbital wavefunction
 $\Rightarrow J = S = \frac{3}{2}$



Therefore, the wavefunction
 is totally symmetric!

$$|\Delta^{++}\rangle \propto |u\uparrow u\uparrow u\uparrow\rangle \times (\text{symmetric spatial piece})$$

But, Δ^{++} is a fermion \Rightarrow wavefunction needs to
 be antisymmetric!

Accepted solution: There exists a new degree of freedom, color. Each flavor of quark comes in 3 colors, which form the $\underline{3}$ of $SU(3)_c$.
 N.B. $SU(3)_c \neq SU(3)_F$.

It is currently believed that $SU(3)_c$ is an exact symmetry of nature.

Nomenclature: **RGB** \rightarrow White

Postulate: Hadrons are color singlets of $SU(3)_c$
 \Rightarrow they sit in the $\underline{1}$ of $SU(3)_c$

This is Color Confinement

For example, baryons qqq : $\square \times \square \times \square \supset \underline{1} + \dots$

\Rightarrow Color wave function

$$\underline{1} = \frac{1}{\sqrt{6}} (RGB - GRB + \text{cyclic perm.})$$

This fixes Δ^{++} because now we need spin \times flavor \times spatial to combine with color.

There is theoretical support for color.

Consider the theoretical calculation of decay rate

$\pi^0 \rightarrow 2\gamma$. One can show that

$$\Gamma(\pi^0 \rightarrow 2\gamma) \propto \left| \sum_i I_{3,2} Q_i^2 \right|^2$$

↑ ↑ ↑
quarks isospin charge

Let N_c = number of quark colors. One finds

$$\begin{aligned} |\dots|^2 &= \left| N_c \left[\left(\frac{1}{2}\right) \left(+\frac{2}{3}\right)^2 + \left(-\frac{1}{2}\right) \left(-\frac{1}{3}\right)^2 + 0 \right] \right|^2 \\ &= \frac{N_c^2}{36} \end{aligned}$$

experimentally rate gives $N_c = 3$.

Another example is the R-ratio,

$$\begin{aligned} R &= \frac{\sigma(e^-e^+ \rightarrow \text{hadrons})}{\sigma(e^-e^+ \rightarrow \mu^-\mu^+)} \propto \sum_i Q_i^2 \\ &= N_c \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = \frac{2}{3} N_c \end{aligned}$$

experiment gives $R = 2 \Rightarrow N_c = 3$.

The quark model gives insight into the hadron structure of the hadron zoo. However, there are severe issues with dynamical quark models and the hadron spectrum. Quarks themselves are never observed, they are permanently confined into color-neutral hadrons, however there is "indirect" evidence for their existence.

Due to the success of QED, we might expect to formulate a QFT of strong interactions based on quarks. As we will see, promoting the color group $SU(3)_c$ gives rise to Quantum Chromodynamics (QCD), which is a gauge theory of quarks and gluons, the gauge field associated w/ $SU(3)_c$. It is QCD that is currently the accepted theory of strong interactions. We will see that many features of the quark model will carry over, and is useful in constructing operators for non-perturbative studies of hadrons with Lattice QCD.