The Standard Model of Particle Physics

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Introduction

The Standard Model (SM) of Particle Physics is the most successful theory of nature. Presently, it accurately describes all experimental measurements including Strong, Weak, & Electromagnetic interactions. This course aims to demonstrate how it is realized in nature.

The Standard Model of Particle Physics is an anomaly-free, renormalizable, relativistic Quantum Field Theory which is invariant under the local gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

which spontaneously breaks via a scalar field to $SU(3)_c \times U(1)_Q$

Our goal is to construct the SM and illustrate its success by studying select predictions and experiments.

We begin by defining the SM in terms of its local gauge and global symmetries and its elementary particle content (spin-half lepton & quarks, spin-one gauge bosons, and a spin-zero Higgs boson). The discrete symmetries (parity P, charge conjugation C, and time reversal T) of the theory are investigated. We show that these symmetries need not manifest in nature, e.g., only left-handed particles feel the weak force, violating parity symmetry.

Spontaneous symmetry breaking, or "hidden symmetry", is vital in understanding the Higgs mechanism of the $SU(2)_L \times U(1)_Y$ electroweak gauge theory and the weakness of the Weak force.

We show how to obtain cross-sections and decay rates from QFT matrix elements. We employ perturbation theory to compute scattering and decay processes for the Electroweak sector, such as Weak decays and neutral meson oscillations.

We will touch upon the topic of neutrino oscillations and masses, which is an important aspects of physics beyond the SM.

The strong interaction is described by Quantum Chromodynamics (QCD), the non-abelian gauge theory of (unbroken) SU(3) gauge symmetry. At low-energies, quarks are confined and form bound states called <u>hadrons.</u> As the energy scale increases, the strong coupling decreases such that perturbation theory can be used. We will look at cases such as the high-energy production of hadrons from electron-positron annihilation, and the structure of the proton via deep inelastic scattering.

Time permitting, we will discuss non-perturbative approaches to QCD. For example, the framework of effective field theories can be used to make progress in the limits of very small and very large quark masses. Numerical techniques such as Lattice QCD can be used to probe the low-energy region of QCD.

Both very high-energy experiments and very precise experiments are currently striving to observe effects that cannot be described by the SM alone. If time permits, we will take a look into how the SM is treated as an effective field theory to accomodate (so far hypothetical) effects beyond the SM.

Overview of the Standard Model

Our current view of the universe is we live in a 4-dimensional spacetime in which all phenomena can be described by a relatively small number of particles that interact via a few well-defined laws.

There are four known forces, Electromagnetism, Weak, Strong, and Gravitational. All forces except Gravitation are described by the SM. From here on out, we will not consider Gravity.

All of the SM forces are mediated by gauge bosons.

Mediators

- photon (γ)
    - Electromagnetic interaction
- Intermediate Vector Bosons ($W^{\pm}, Z^{0}$)
    - Weak Interaction
- Gluons (g)
    - Strong Interaction
    - There are 8 gluons in the SM

Property	photon (γ)	I.V.B. ($W^{\pm}, Z^0$)	gluons (g)
Spin	1	1	1
mass	0	$\sim 80, 90$ GeV	0
charge	0	$\pm 1, 0$	0
Coupling strength	$\sim 10^{-2}$	$\sim 10^{-6}$	~ 1
range	∞	$\sim 10^{-2}$ fm	$\sim 10^{-1}$ fm
Sources affected	charged	all	Quarks (Hadrons)
mediators affected	$W^{\pm}, Z^0$	$\gamma, W^{\pm}, Z^0$	g
expt. status	seen	seen	not seen directly
typical process decay time	$\sim 10^{-18}$ s	$\sim 10^{-9}$ s	$\sim 10^{-23}$ s
typical cross section	$\sim 10^{-36}$ m^2	$\sim 10^{-42}$ m^2	$\sim 10^{-32}$ m^2

While electromagnetic and weak interactions appear to be very different, they can be partially unified into the electroweak interaction.

All matter interacts with each other via the gauge bosons. Matter is described by spin-$1/2$ fermions. These are described by Dirac spinors. We can classify them into 4 types, and each type comes in 3 generations.

There are two classes of sources - Leptons and Quarks

Leptons	G_1	G_2	G_3	charge
Charged leptons	e	μ	τ	-1
Neutral leptons (Neutrinos)	ν_e	ν_μ	ν_τ	0

All leptons are spin-$1/2$ fermions. The charged leptons are massive, and the neutrinos are almost massless. The minimal SM gives the neutrinos as massless, but we have observed that they do indeed have a mass.

The leptons do not interact with the strong interaction

	EM	Weak	Strong
Charged leptons	✓	✓	✗
neutrinos	✗	✓	✗

Quarks	G_1	G_2	G_3	Charge
positive quarks	u	c	t	$+2/3$
negative quarks	d	s	b	$-1/3$

All quarks are massive spin-$\frac{1}{2}$ fermions. The different quark types are called Flavors. The quarks have another quantum number, Color. There are three colors:

Red, Green, Blue (R, G, B)

Therefore, there are $3 \times 6 = 18$ quarks. The quarks are not directly observed. We only experimentally detect combinations of quarks called hadrons

Hadrons

Baryons	Mesons	Exotica*
- 3 valence q's	- valence $q + \bar{q}$	- tetraquarks $(2q + 2\bar{q})$
- spin-$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots$	- spin-0, 1, 2, ...	- Pentaquarks $(4q + \bar{q})$
- $p, n, \Delta, \dots$	- $\pi, K, \rho, \omega, \dots$	- Hybrids $(q\bar{q} + g)$
		- Glueballs
		...

* Exotica refers to Non-Quark model objects

Quarks interact with all SM forces,

	EM	Weak	Strong
U-type quarks	✓	✓	✓
D-type quarks	✓	✓	✓

We do not know why there are three generations of matter. Note that the total charge of a generation sums to zero,

$$Q_{G_i} = \underset{\substack{\uparrow \\ \text{lepton}}}{-1} + \underset{\substack{\uparrow \\ \text{neutrino}}}{0} + \underset{\substack{\uparrow \\ \text{3 colors}}}{3} \times \left(\underset{\substack{\uparrow \\ \text{up-type}}}{\frac{2}{3}} - \underset{\substack{\uparrow \\ \text{down-type}}}{\frac{1}{3}} \right) = 0 \qquad \forall\, i = 1, 2, 3$$

In addition to these particles, there exists their antiparticles, in which all quantum numbers are negated, e.g., $e^+ = \bar{e}$ $(Q = +1)$

$\bar{u}$ $(Q = -2/3,\ \bar{R}, \bar{B}, \bar{G})$

Finally, there is the Higgs boson, which is a spin-0, massive, and neutral particle. The Higgs is responsible for giving mass to the $W^\pm$, Z^0 bosons and the fermions.

All of the SM forces are described by Gauge theories. Gauge theories are Quantum field theories which are invariant under gauge transformations. The forces are mediated by spin-1 gauge bosons.

The gauge group of the SM is

$$\boxed{\underset{\substack{\uparrow \\ \text{Strong interaction}}}{SU(3)_c} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{Electroweak interaction}}}$$

c = Color

L = Left-handed (chiral)

Y = Weak hypercharge

The Higgs field spontaneously breaks the Electroweak symmetry group

$$SU(2)_L \times U(1)_Y \longrightarrow \underset{\substack{\uparrow \\ \text{Electromagnetic interaction}}}{U(1)_Q}$$

Q = electric charge

Matter fields transform in distinct representations of the gauge group. Those representations dictate interactions between the matter fields and the gauge fields.

The SM is a chiral theory, which means that left-handed and right handed fields interact differently. The SM matter fields can be collected as follows:

$$\begin{array}{c} \times 3 \\ \text{generations} \end{array} \left\{ \begin{array}{lll} Q_L & (\underset{\sim}{3},\underset{\sim}{2})_{\frac{1}{3}} & \text{Left-handed Quarks} \\ U_R & (\underset{\sim}{3},\underset{\sim}{1})_{4/3} & \text{Right-handed up-Quarks} \\ D_R & (\underset{\sim}{3},\underset{\sim}{1})_{-2/3} & \text{Right-handed down-Quarks} \\ L_L & (\underset{\sim}{1},\underset{\sim}{2})_{-1} & \text{Left-handed Leptons} \\ E_R & (\underset{\sim}{1},\underset{\sim}{1})_{-2} & \text{Right-handed charged leptons} \end{array} \right.$$

$$? \times 3 \qquad N_R \quad (\underset{\sim}{1},\underset{\sim}{1})_0 \qquad \text{Right-handed neutrinos ?}$$

The notation denotes the dimension of the corresponding $SU(3)_c$ and $SU(2)_L$ representations, and the subscript the $U(1)_Y$ hypercharge.

Example,

$$Q_L(\underset{\sim}{3}, \underset{\sim}{2})_{1/3} \leftarrow \text{hypercharge } Y = 1/3$$

SU(3)$_c$ triplet SU(2) doublet

The Higgs fields are scalar fields $\phi(\underset{\sim}{1}, \underset{\sim}{2})_1$.

This is the field content of the SM.

Types of Symmetry

As one could imagine, a key principle in the study of the SM is symmetry. Symmetries can manifest themselves in a number of ways.

(i) We have intact or exact symmetries. These are actual symmetries of the theory. For example, $U(1)_Q$ and $SU(3)_c$ are exact symmetries of the SM.

(ii) Symmetries can be broken by an anomaly. This is a symmetry that exists in the classical theory but is broken when we quantize. An important example is the global axial anomaly for massless spinor fields in the SM. However, by coincidence, the SM is anomaly-free due to a cancellation between the field content of the SM.

(iii) Symmetry is explicitly broken by some terms in the Lagrangian. This is not a symmetry, but if the terms are small, then we have an approximate symmetry, and it is sometimes useful to consider these.

For example, the up and down quarks are very close in mass, but not exactly the same. This gives an approximate global isospin symmetry.

(iv) The symmetry is respected by the Lagrange density $\mathcal{L}$, but not by the vacuum. This is a hidden symmetry.

(a) We can have a spontaneously broken symmetry. We have a vacuum expectation value for one or more scalar fields.
For example, the breaking of

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$$

in the Standard Model.

(b) Even without scalar fields, we can get dynamical symmetry breaking from quantum effects. An example of this in the Standard Model is the $SU(2)_L \times SU(2)_R$ global symmetry in the strong interaction.

Only (i) is an actual symmetry, but the others are very useful to consider in our study of the Standard Model.