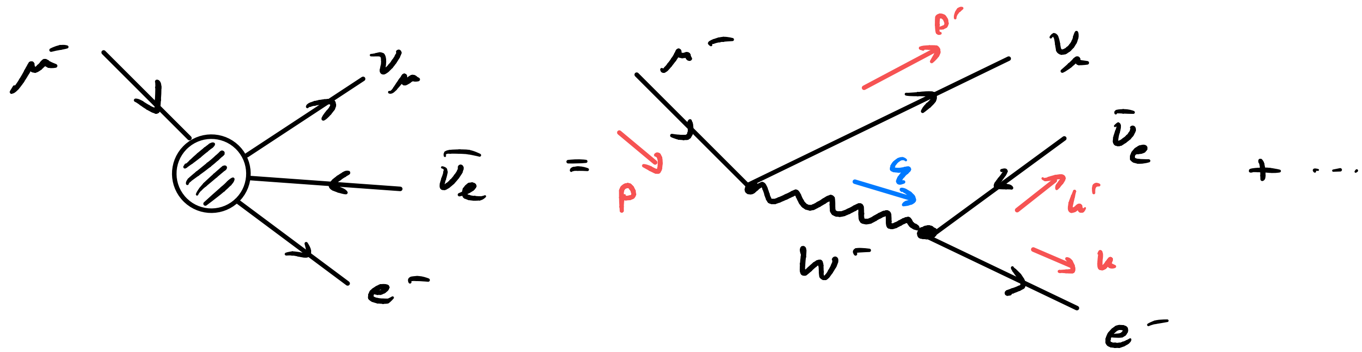
Phenomenology III – Electroweak Interactions

Let us explore some low-energy phenomenology from the electroweak theory of leptons. For example, consider μ decay, $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. At leading order in "$g$", the amplitude is



$$i\mathcal{M} = \bar{u}_e\left(-\frac{ig}{\sqrt{2}}\gamma_\mu P_L\right)v_{\nu_e}\left(\frac{-i}{q^2-m_W^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{m_W^2}\right)\right)\bar{u}_{\nu_\mu}\left(\frac{-ig}{\sqrt{2}}\gamma_\nu P_L\right)u_\mu$$

The muon mass $m_\mu \ll m_W$, so let us construct an effective interaction by taking $q^2 \ll m_W^2$

So,

$$\frac{1}{q^2-m_W^2} = -\frac{1}{m_W^2}\left(\frac{1}{1-q^2/m_W^2}\right) = -\frac{1}{m_W^2} + \mathcal{O}\left(\frac{q^2}{m_W^4}\right)$$

$$\Rightarrow \quad i\mathcal{M} = -\frac{ig^2}{8m_W^2}\,\bar{u}_e\gamma_\mu(1-\gamma_5)v_{\nu_e}\,\bar{u}_{\nu_\mu}\gamma^\mu(1-\gamma_5)u_\mu + \mathcal{O}\left(\frac{q^2}{m_W^4}\right)$$

Define Fermi decay constant G_F, determined from τ_{μ^-}

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} \simeq \frac{1}{\sqrt{2}}\left(1.166\times10^{-5}\ \mathrm{GeV}^{-2}\right)}$$

This effective interaction is known as the <u>four-Fermi interaction</u>, which was historically developed first to describe neutron β-decay.

We want to compute the decay rate in the muon rest frame,

$$d\Gamma = \frac{1}{2m_\mu} \langle |\mathcal{M}|^2 \rangle (2\pi)^4 \delta^{(4)}(p-p'-k'-k) \frac{d^3\vec{k}}{(2\pi)^3 2E_e} \frac{d^3\vec{k}'}{(2\pi)^3 2E_{\bar{\nu}_e}} \frac{d^3\vec{p}'}{(2\pi)^3 2E_{\nu_\mu}}$$

First, let us compute $\langle |\mathcal{M}|^2 \rangle$

$$\Rightarrow \langle |\mathcal{M}|^2 \rangle = \frac{1}{2} \sum_s \sum_{s',r,r'} |\mathcal{M}|^2$$

Now,

$$\Gamma^\alpha = \gamma^\alpha(1-\gamma_5)$$

$$\begin{aligned}
|\mathcal{M}|^2 &= \mathcal{M}^+ \mathcal{M} \\
&= \frac{G_F^2}{2} \left(\bar{v}_{\nu_e} \Gamma^\alpha u_e\right)\left(\bar{u}_\mu \Gamma_\alpha u_{\nu_\mu}\right)\left(\bar{u}_e \Gamma_\beta v_{\nu_e}\right)\left(\bar{u}_{\nu_\mu} \Gamma^\beta u_\mu\right) \\
&= \frac{G_F^2}{2} \mathrm{tr}\left[\bar{v}_{\nu_e} \Gamma^\alpha u_e \bar{u}_e \Gamma_\beta v_{\nu_e}\right] \mathrm{tr}\left[\bar{u}_\mu \Gamma_\alpha u_{\nu_\mu} \bar{u}_{\nu_\mu} \Gamma^\beta u_\mu\right]
\end{aligned}$$

$$\Rightarrow \langle |\mathcal{M}|^2 \rangle = \frac{G_F^2}{4} \mathrm{tr}\left(\not{k}' \Gamma^\alpha (\not{k}+m_e) \Gamma_\beta\right) \mathrm{tr}\left((\not{p}+m_\mu)\Gamma_\alpha \not{p}' \Gamma^\beta\right)$$

Evaluating the traces, e.g., with Mathematica, we find the following,

$$\mathrm{tr}\left(\not{k}' \Gamma^{\alpha} (\not{k} + m_e) \Gamma_{\rho} \right)$$
$$= 8 \left(k^{\rho} k'^{\alpha} + k^{\alpha} k'^{\rho} - (k \cdot k') g^{\alpha\rho} + i \epsilon^{\alpha\rho\mu\nu} k_{\mu} k'_{\nu} \right)$$

$$\mathrm{tr}\left((\not{p} + m_{\mu}) \Gamma_{\alpha} \not{p}' \Gamma^{\rho} \right)$$
$$= 8 \left(p^{\rho} p'^{\alpha} + p^{\alpha} p'^{\rho} - (p \cdot p') g^{\alpha\rho} - i \epsilon^{\alpha\rho\mu\nu} p_{\mu} p'_{\nu} \right.$$

Contracting these elements, and using the antisymmetry properties of $\epsilon^{\mu\nu\rho\sigma}$, we find,

$$\langle |\mathcal{M}|^2 \rangle = 64\, G_F^2 \, (p \cdot k')(k \cdot p')$$

Thus, the decay rate is

$$\Gamma = \frac{1}{2 m_{\mu}} \int \frac{d^3 \vec{k}}{(2\pi)^3 2E_e} \frac{d^3 \vec{k}'}{(2\pi)^3 2E_{\nu_e}} \frac{d^3 \vec{p}'}{(2\pi)^3 2E_{\bar{\nu}_{\mu}}} (2\pi)^4 \delta^{(4)}(p - p' - k' - k) \langle |\mathcal{M}|^2 \rangle$$

$$= \frac{64 G_F^2 (2\pi)^4}{2^4 (2\pi)^9 m_{\mu}} \int \frac{d^3 \vec{k}\, d^3 \vec{k}'\, d^3 \vec{p}'}{E_e E_{\bar{\nu}_e} E_{\nu_{\mu}}} \delta^{(4)}(p - p' - k' - k)\, (p \cdot k')(k \cdot p')$$

$$= \frac{G_F^2}{8\pi^5 m_{\mu}} \int \frac{d^3 \vec{k}}{E_e} \int \frac{d^3 \vec{k}'\, d^3 \vec{p}'}{|\vec{k}'|\,|\vec{p}'|} \delta^{(4)}(p - k - p' - k')\, (p \cdot k')(k \cdot p')$$

$E_{\bar{\nu}_e} = |\vec{k}'|$, $E_{\nu_{\mu}} = |\vec{p}'|$ since $m_{\nu} = 0$

Let's focus on the $\bar{\nu}_e - \nu_\mu$ subsystem.

Let $Q = p - k$, and define

$$I_{\mu\nu}(Q) \equiv \int \frac{d^3\vec{k}'\, d^3\vec{p}'}{|\vec{k}'|\,|\vec{p}'|}\, \delta^{(4)}(Q - k' - p')\, k'_\mu\, p'_\nu$$

From Lorentz covariance,

$$I_{\mu\nu}(Q) = \underbrace{A(Q^2)\, Q_\mu Q_\nu + B(Q^2)\, g_{\mu\nu} Q^2}_{\text{Real scalars}}$$

so,

$$g^{\mu\nu} I_{\mu\nu} = (A + 4B)\, Q^2$$

and

$$g^{\mu\nu} I_{\mu\nu} = \int \frac{d^3\vec{k}'\, d^3\vec{p}'}{|\vec{k}'|\,|\vec{p}'|}\, \delta^{(4)}(Q - k' - p')\, k' \cdot p'$$

But,

$$(k' + p')^2 = \underbrace{k'^2 + p'^2}_{\text{Massless neutrinos}} + 2k' \cdot p' = 2k' \cdot p'$$

also, from momentum conservation

$$p - k = p' + k' = Q$$

so,

$$\Rightarrow k' \cdot p' = \frac{1}{2} Q^2$$

$$\Rightarrow A + 4B = \frac{I}{2} \equiv \frac{1}{2} \int \frac{d^3\vec{k}'\, d^3\vec{p}'}{|\vec{k}'|\,|\vec{p}'|}\, \delta^{(4)}(Q - k' - p')$$

Consider instead,

$$Q^\mu Q^\nu I_{\mu\nu} = (A + B) Q^4$$
$$= \int \frac{d^3\vec{k}' \, d^3\vec{p}'}{|\vec{k}'| |\vec{p}'|} \delta^{(4)}(Q - k' - p') (k' \cdot Q)(p' \cdot Q)$$

But, $Q = p - k = p' + k'$

$$\Rightarrow k' \cdot Q = k' \cdot p' \quad \text{since } k'^2 = 0$$
$$p' \cdot Q = k' \cdot p' \quad \text{since } p'^2 = 0$$

and $(k' \cdot p')^2 = \frac{1}{4} Q^4$

$$\Rightarrow A + B = \frac{I}{4}$$

$$\Rightarrow \left.\begin{aligned} A + 4B &= \frac{1}{2} I \\ A + B &= \frac{1}{4} I \end{aligned}\right\} \Rightarrow \begin{aligned} A &= \frac{1}{6} I \\ B &= \frac{1}{12} I \end{aligned}$$

So, computing I

$$I = \int \frac{d^3\vec{k}' \, d^3\vec{p}'}{|\vec{k}'| |\vec{p}'|} \delta^{(4)}(Q - k' - p')$$
$$= \int \frac{d^3\vec{k}'}{|\vec{k}'|} \frac{d^3\vec{p}'}{|\vec{p}'|} \delta(Q^0 - |\vec{k}'| - |\vec{p}'|) \, \delta^{(3)}(\vec{Q} - \vec{k}' - \vec{p}')$$

↳ choose frame where $\vec{Q} = \vec{0} \Rightarrow \vec{k}' = -\vec{p}'$

$$= \int \frac{d^3\vec{k}'}{|\vec{k}'|^2} \delta(Q^0 - 2|\vec{k}'|)$$

$\rightarrow Q^0 = \sqrt{Q^2}$

So,
$$I = \int \frac{d^3\vec{k}'}{|\vec{k}'|^2}\, \delta(Q^0 - 2|\vec{k}'|)$$
$$= 4\pi \int_0^\infty d|\vec{k}'|\, \delta(Q^0 - 2|\vec{k}'|)$$
$$= 2\pi \int_0^\infty d|\vec{k}'|\, \delta(|\vec{k}'| - Q^0/2)$$
$$= 2\pi$$

So,
$$I_{\mu\nu} = \frac{2\pi}{6} Q_\mu Q_\nu + \frac{2\pi}{12} Q^2 g_{\mu\nu}$$
$$= \frac{\pi}{6}\left(2Q_\mu Q_\nu + Q^2 g_{\mu\nu}\right), \quad Q_\mu = p_\mu - k_\mu$$

The Decay rate is then,
$$\Gamma = \frac{G_F^2}{8\pi^5 m_\mu} \int \frac{d^3\vec{k}}{E_e} \int \frac{d^3\vec{k}'\, d^3\vec{p}'}{|\vec{k}'|\,|\vec{p}'|}\, \delta^{(4)}(p - k - p' - k')\,(p\cdot k')(k\cdot p')$$
$$= \frac{G_F^2}{8\pi^5 m_\mu} \int \frac{d^3\vec{k}}{E_e}\, I_{\mu\nu}\, p^\mu k^\nu$$
$$= \frac{G_F^2}{48\pi^4 m_\mu} \int \frac{d^3\vec{k}}{E_e}\left(2(p-k)\cdot p\,(p-k)\cdot k + (p-k)^2\, p\cdot k\right)$$

In the rest frame of μ, $p^2 = m_\mu^2$, $k^2 = m_e^2$
$$p\cdot k = E_\mu E_e = m_\mu E_e$$

Now, $\frac{m_e}{m_\mu} \simeq 0.0048 \ll 1$

$\Rightarrow$ Assume $m_e = 0 \Rightarrow E_e = |\vec{k}|$

therefore,

$$\Gamma = \frac{G_F^2}{48\pi^4 m_\mu} \int \frac{d^3\vec{k}}{E_e} \left(3 m_\mu^3 E_e - 4(m_\mu E_e)^2\right)$$

$$= \frac{G_F^2 m_\mu}{48\pi^4} \int d^3\vec{k} \, (3 m_\mu - 4 E_e)$$

$$= \frac{G_F^2 m_\mu}{48\pi^4} \cdot 4\pi \int dE_e \, E_e^2 (3 m_\mu - 4 E_e)$$

What are the bounds for E_e?

Minimum is when $E_{\text{kin}} = 0$ (electron at "rest")

Maximum is when $\bar{\nu}_e, \nu_\mu$ are collinear, opposite e^-.

So, Energy conservation $\Rightarrow E_e + E_{\bar{\nu}_e} + E_{\nu_\mu} = m_\mu$

Momentum conservation $\Rightarrow E_e - (E_{\bar{\nu}_e} + E_{\nu_\mu}) = 0$

So, $E_{max} = \frac{m_\mu}{2}$

$$\Rightarrow \Gamma = \frac{G_F^2 m_\mu}{12\pi^3} \int_0^{m_\mu/2} dE_e \, E_e^2 (3 m_\mu - 4 E_e) = \boxed{\frac{G_F^2 m_\mu^5}{192\pi^3}}$$

The $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ is the dominant decay mode, BR ~ 100 %. So, we can measure the lifetime,

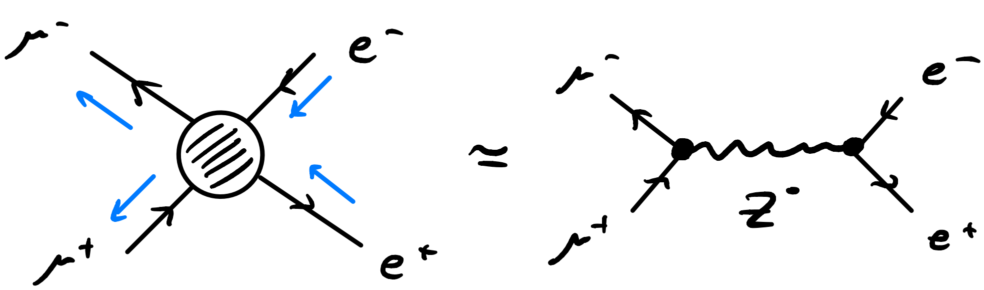
$$\tau_\mu \simeq 2.1870 \times 10^{-6}\ \mathrm{s}$$

and deduce that $\boxed{G_F \simeq 1.164 \times 10^{-5}\ \mathrm{GeV}^{2}}$

we find that $\tau \to e \bar{\nu}_e \nu_\tau$ is consistent with this G_F. $\Rightarrow$ lepton universality

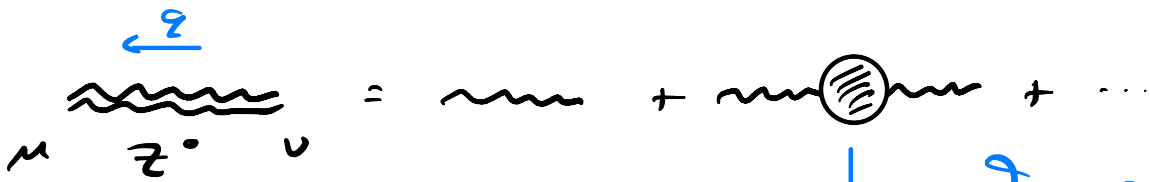
Z-Boson Phenomenology

The Z-boson is unstable, and thus we cannot detect it directly. However, we can learn about it as a resonance in leptonic reactions, e.g., $e^- e^+ \to \mu^- \mu^+$



Unstable particles have a decay width, and thus are poles in the complex energy plane.

Consider the Dyson series for the Z-boson propagator



↳ quantum corrections $i\Pi(q^2)$

Dressed propagator

$$= \cdots$$

$$= \frac{1}{q^2 - m_Z^2 + \Pi(q^2)} \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_Z^2} \right)$$

↳ parameter, not Z-boson mass!

$$\equiv D(q^2) \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_Z^2} \right)$$

The physical Z-boson mass is the real part of the pole of the propagator. Let m_Z^R be the physical resonant pole mass, & Γ_Z^R the physical decay width.

Let's expand $\mathrm{Re}\,\Pi(q^2)$ about $q^2 = m_Z^{R\,2}$

$$\mathrm{Re}\,\Pi(q^2) = \mathrm{Re}\,\Pi(m_Z^{R\,2}) + \left.\frac{d\,\mathrm{Re}\,\Pi}{dq^2}\right|_{q^2 = m_Z^{R\,2}} (q^2 - m_Z^{R\,2}) + \cdots$$

So

$$D(q^2) = \frac{1}{q^2 - m_Z^2 + \mathrm{Re}\,\Pi(q^2) + i\,\mathrm{Im}\,\Pi(q^2)}$$

$$= \frac{1}{q^2 - m_Z^2 + \mathrm{Re}\,\Pi(m_Z^{R\,2}) + (q^2 - m_Z^{R\,2})\,\mathrm{Re}\,\Pi'(m_Z^{R\,2}) + i\,\mathrm{Im}\,\Pi(q^2)}$$

the denominator is

$$\underbrace{q^2 - m_z^2 + \mathrm{Re}\,\Pi(m_z^{R\,2})}_{m_z^{R\,2} = m_z^2 - \mathrm{Re}\,\Pi(m_z^{R\,2})} + (q^2 - m_z^{R\,2})\,\mathrm{Re}\,\Pi'(m_z^{R\,2}) + \cdots + i\,\mathrm{Im}\,\Pi(q^2)$$

$$= (q^2 - m_z^{R\,2})\underbrace{\left[1 + \mathrm{Re}\,\Pi'(m_z^{R\,2}) + \cdots\right]}_{\equiv Z^{-1}} + i\,\mathrm{Im}\,\Pi(q^2)$$

So,

$$iD(q^2) = \frac{Z}{q^2 - m_z^{R\,2} + i\,Z\,\mathrm{Im}\,\Pi(q^2)}$$

For stable particles, $\mathrm{Im}\,\Pi(q^2) = 0 \Rightarrow q^2 = m_z^{R\,2}$ pole!

BUT, for an unstable particle, $\mathrm{Im}\,\Pi(q^2) \neq 0$

So, near $q^2 = m_z^{R\,2}$, $Z\,\mathrm{Im}\,\Pi(m_z^{R\,2}) \equiv m_z^R \Gamma_z^R$

So, as $q^2 \sim m_z^{R\,2}$, $iD(q^2) \sim \dfrac{Z}{q^2 - (m_z^{R\,2} - i m_z^R \Gamma_z^R)}$

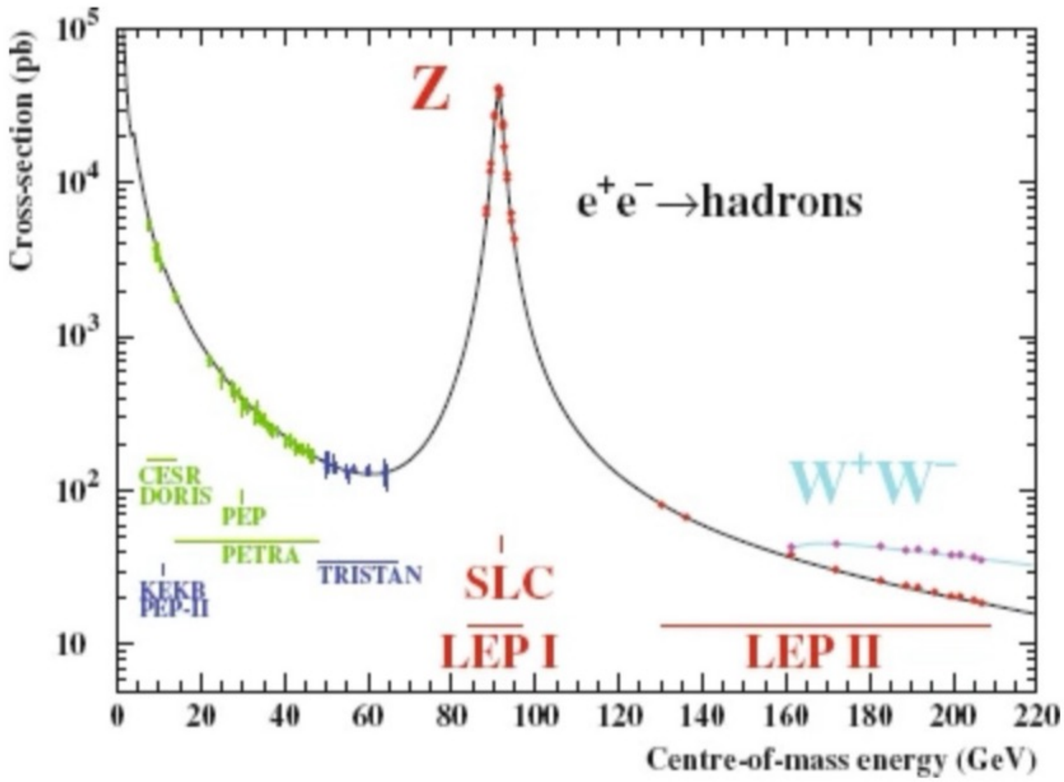
if $\dfrac{\Gamma_z^R}{m_z^R} \ll 1$, $\Rightarrow q^2 = m_z^{R\,2} - i m_z^R \Gamma_z^R$

$$\simeq \left(m_z^R - \tfrac{i}{2}\Gamma_z^R\right)^2$$

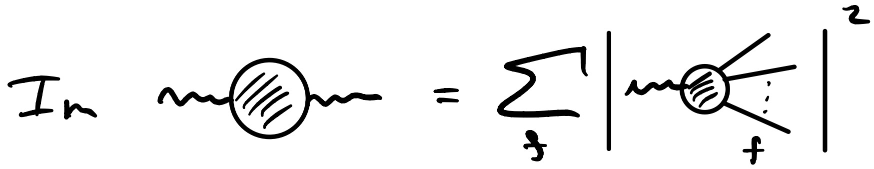
This is the <u>relativistic Breit-Wigner amplitude</u>.

One can show that near the Z-boson mass, the cross-section for $e^-e^+ \to \mu^-\mu^+$ is

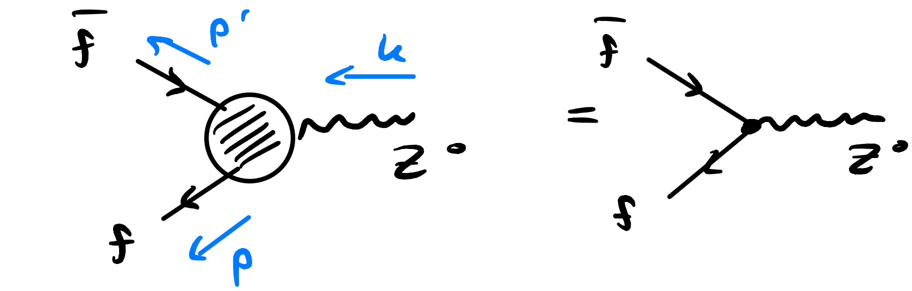
$$\sigma \simeq \frac{4\pi\alpha^2}{3s}\left[1 + \frac{1}{16\sin^2 2\theta_W} \frac{s^2}{\left(s - m_Z^2 + \frac{\Gamma_Z^2}{4}\right)^2 + m_Z^2\Gamma_Z^2} \right]$$



The optical theorem relates $\mathrm{Im}\,\Pi(q^2) \leftrightarrow \Gamma_Z$



Let's compute the decay rate for $Z \to f\bar{f}$, $f = \ell, \nu$ at leading order.



$$\Rightarrow \quad i\mathcal{M} = \bar{u}(p,s)\left[-i\frac{g}{\cos\theta_W}\gamma^\mu\left(\frac{1}{2}T_3 - Q_f\sin^2\theta_W - \frac{1}{2}T_3\gamma_5\right)\right]v(p',s')\,\epsilon_\mu(k,\lambda)$$

$$= -i\frac{g}{\cos\theta_W}\,\bar{u}(p,s)\gamma^\mu(v_f - a_f\gamma_5)v(p',s')\,\epsilon_\mu(k,\lambda)$$

$\hookrightarrow$ Z-boson polarization

So,
$$\Gamma_Z = \frac{|\vec{p}|}{32\pi^2 m_Z^2}\int d\Omega\,\frac{1}{3}\sum_{s',s,\lambda}|\mathcal{M}|^2$$

$\hookrightarrow$ average over initial polarizations

Now,
$$|\mathcal{M}|^2 = \frac{g^2}{\cos^2\theta_W}\,\bar{u}(p,s)\gamma^\mu(v_f - a_f\gamma_5)v(p',s')\,\bar{v}(p',s')\gamma^\nu(v_f - a_f\gamma_5)u(p,s)$$
$$\times\ \epsilon_\mu(k,\lambda)\,\epsilon_\nu^*(k,\lambda)$$

Using
$$\sum_s u(p,s)\bar{u}(p,s) = p^2 + m_f^2$$

$$\sum_{s'} v(p',s')\bar{v}(p',s') = p'^2 - m_f^2$$

$$\sum_\lambda \epsilon_\mu(k,\lambda)\epsilon_\nu^*(k,\lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2}$$

we find

$$\sum_{s,s',\lambda} |\mathcal{M}|^2 = \frac{g^2}{\cos^2\theta_W}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2}\right)$$

$$\times\ \mathrm{tr}\left[(\not{p}+m_f)\gamma^\mu(v_f - a_f\gamma_5)(\not{p}'-m_f)\gamma^\nu(v_f - a_f\gamma_5)\right]$$

Let use that $\frac{m_{f=e,\nu}}{m_Z} \ll 1$

$$\Rightarrow \sum_{s,s',\lambda} |\mathcal{M}|^2 \simeq \frac{g^2}{\cos^2\theta_W}\left(-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2}\right)\mathrm{tr}\left[\not{p}\gamma^\mu(v_f - a_f\gamma_5)\not{p}'\gamma^\nu(v_f - a_f\gamma_5)\right]$$

$$= \frac{g^2}{\cos^2\theta_W}\cdot 8(a_f^2+v_f^2)\left(\frac{(k\cdot p)(k\cdot p')}{m_Z^2} + (p\cdot p') - \frac{k^2(p\cdot p')}{2m_Z^2}\right)$$

$$= \frac{8g^2(a_f^2+v_f^2)}{\cos^2\theta_W}\left(\frac{(k\cdot p)(k\cdot p')}{m_Z^2} + \frac{1}{2}(p\cdot p')\right) \quad k^2 = m_Z^2$$

In the rest frame of the Z^0-boson, $k = (m_Z, \vec{0})$

$$\vec{p} = -\vec{p}' \Rightarrow E' = E = |\vec{p}| = \frac{m_Z}{2}$$

$$\Rightarrow k\cdot p = k\cdot p' = m_Z E = \frac{m_Z^2}{2}$$

$$m_Z^2 = (p+p')^2 = p^2 + p'^2 + 2p'\cdot p \Rightarrow p'\cdot p = \frac{m_Z^2}{2}$$

$$\Rightarrow \sum_{s',s,\lambda} |\mathcal{M}|^2 = \frac{4g^2 m_Z^2}{\cos^2\theta_W}(a_f^2 + v_f^2)$$

$$\xrightarrow{} |\vec{p}| = \frac{m_Z}{2}$$

So,
$$\Gamma_Z = \frac{|\vec{p}|}{32\pi^2 m_Z^2} \int d\Omega \, \frac{1}{3} \sum_{s',s,\lambda} |\mathcal{M}|^2$$

$$= \frac{1}{64\pi^2 m_Z} \cdot \frac{4\pi}{3} \cdot \frac{4g^2 m_Z^2}{\cos^2\theta_W} (a_f^2 + v_f^2)$$

$$= \frac{g^2 m_Z}{12\pi \cos^2\theta_W} (a_f^2 + v_f^2)$$

In terms of the fermi constant G_F, $g^2 = \frac{8 G_F m_W^2}{\sqrt{2}}$

$$\Gamma_Z = \frac{2}{3\sqrt{2}\pi} \frac{G_F m_W^2 m_Z}{\cos^2\theta_W} (a_f^2 + v_f^2)$$

Also $m_W = m_Z \cos\theta_W \Rightarrow m_W^2 = m_Z^2 \cos^2\theta_W$

$$\Rightarrow \Gamma_Z = \frac{2}{3\sqrt{2}\pi} G_F m_Z^3 (a_f^2 + v_f^2)$$

- For neutrinos, $a_f^2 = v_f^2 = \frac{1}{16}$

$$\Rightarrow \boxed{\Gamma(Z^\circ \to \nu\bar{\nu}) = \frac{1}{12\sqrt{2}\pi} G_F m_Z^3}$$

$$\simeq 167 \text{ MeV}$$

- For charged leptons, $a_f^2 = \frac{1}{16}$, $v_f = \left(-\frac{1}{4} + \sin^2\theta_W\right)^2$

$$= \frac{1}{16}\left(1 - 4\sin^2\theta_W\right)^2 \equiv \frac{g_V^2}{16}$$

So,

$$\boxed{\Gamma(Z^\circ \to \ell\bar{\ell}) = \frac{1}{24\sqrt{2}\pi} G_F \left(1+g_V^2\right) m_Z^3}$$

$$\simeq 28 \text{ MeV}$$

Therefore, we find for 3 generations of leptons, $\ell = e, \mu, \tau$, that

$$\Gamma(Z^\circ \to \text{leptons}) \simeq 501 \text{ MeV}$$

The fact that the neutrinos give identical contribution to the decay width makes it a useful measure to determine the number of "massless" neutrinos. Precision measurements give

$$\Gamma^{(exp)}(Z^\circ \to \text{hadrons}) = 1748 \pm 35 \text{ MeV}$$

$$\Gamma^{(exp)}(Z^\circ \to \ell\bar{\ell}) = 83 \pm 2 \text{ MeV}.$$

Therefore, measuring $\Gamma_Z^{(exp)}$, we can deduce the contribution from neutrinos (which are difficult to detect)

$$\Gamma^{(exp)}(Z^\circ \to \nu\bar{\nu}) = \Gamma_Z^{(exp)} - \Gamma^{(exp)}(Z^\circ \to \text{hadrons}) - \Gamma^{(exp)}(Z^\circ \to \ell\bar{\ell})$$

$$= 494 \pm 32 \text{ MeV}.$$

So, number of neutrinos $\Rightarrow$ $$N_\nu = \frac{\Gamma^{(exp)}(Z^\circ \to \nu\bar{\nu})}{\underbrace{\Gamma(Z^\circ \to \nu\bar{\nu})}_{167 \text{ MeV}}} = 2.96 \pm 11$$