The Standard Model of Particle Physics

We have come to the point where we can construct the Standard Model, which is an $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory describing electroweak and strong forces. It is largely based on what has been already discussed, with some modifications. To begin, let's simply add the quarks to the Electroweak Model of leptons, and consider the strong gluon physics afterward.

Electroweak Model of Quarks & Leptons

Recall that the (unbroken) $SU(2)_L \times U(1)_Y$ theory of leptons is

$$
\begin{aligned}
\mathcal{L}_{EW} = & -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\
& + \frac{1}{2} i \bar{L}_A \not{D} L_A + \frac{1}{2} i \bar{R}_A \not{D} R_A + h.c. \\
& + (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!} (\phi^\dagger \phi)^2 \\
& - G^L_A [\bar{R}_A (\phi^\dagger L_A) + (\bar{L}_A \phi) R_A]
\end{aligned}
$$

To add quarks, we must specify how they transform under $SU(2)_L \times U(1)_Y$.

Field	$SU(2)_L$	$U(1)_Y$	T	T_3	Y	$Q = T_3 + \frac{1}{2}Y$
$Q_A = \begin{pmatrix} u_A \\ d_A \end{pmatrix}_L$	$\underset{\sim}{2}$	$1/2$	$1/2$ $1/2$	$1/2$ $-1/2$	$+1/3$ $-1/3$	$+2/3$ $-1/3$
$U_A = u_{AR}$	$\underset{\sim}{1}$	$4/3$	0	0	$+4/3$	$+2/3$
$D_A = d_{AR}$	$\underset{\sim}{1}$	$-2/3$	0	0	$-2/3$	$-1/3$

The covariant derivative is the same as before (but note quantum numbers). We then add the following extra terms to $\mathcal{L}_{EW}$

$$
\begin{aligned}
\mathcal{L}_{EW,\,quarks} = & \frac{1}{2} i \bar{Q}_A \not{D} Q_A + \frac{1}{2} i \bar{U}_A \not{D} U_A + \frac{1}{2} i \bar{D}_A \not{D} D_A + h.c. \\
& - G_A^D \left[\bar{D}_A (\phi^\dagger Q_A) + (\bar{Q}_A \phi) D_A \right] \\
& - G_A^u \left[\bar{U}_A (\phi^{c\dagger} Q_A) + (\bar{Q}_A \phi^c) U_A \right]
\end{aligned}
$$

note!

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \phi^c = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$$

↳ appearance maintains $SU(2)_L \otimes U(1)_Y$

The terms shown are most general, up to possible Yukawa mixings (see soon). No mass terms for the quarks (to preserve $SU(2)_L$) & no cross Yukawa interactions between leptons and quarks.

e.g., $\bar{R}_A (\phi^\dagger Q_A)$ – Not possible under $SU(2)_L \otimes U(1)_Y$!

$Y: \quad +2 \quad -1 \quad +\frac{1}{3} \neq 0$

We previously had 7 parameters, $g, g', \mu, \lambda, G^L_A$,
Now have added 6 more: G^U_A, G^D_A.

Masses for quarks come from SSB, as before

G^D terms: $$\mathcal{L}^D_{Yukawa} = -\frac{1}{\sqrt{2}} G^D_A \left[\bar{d}_{RA} (0, v) \begin{pmatrix} u_{AL} \\ d_{AL} \end{pmatrix} + (\bar{u}_{LA}, \bar{d}_{LA}) \begin{pmatrix} 0 \\ v \end{pmatrix} d_{RA} \right]$$

$$= -\frac{1}{\sqrt{2}} G^D_A v \, \bar{d}_A d_A$$

$$\Rightarrow \boxed{m_{d_A} = \frac{1}{\sqrt{2}} G^D_A v}$$

For G_A^U, need ϕ^c in unitary gauge

$$\phi^c = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$$

So,
$$\mathcal{L}^U_{Yukawa} = -\frac{1}{\sqrt{2}} G_A^U \left[\bar{u}_{R_A} (v, 0) \begin{pmatrix} u_{L_A} \\ d_{L_A} \end{pmatrix} + h.c. \right]$$
$$= -\frac{1}{\sqrt{2}} G_A^U v \, \bar{u}_A u_A$$

$$\Rightarrow \boxed{m_{U_A} = \frac{1}{\sqrt{2}} G_A^U a}$$

For the quarks & leptons, we chose non-mixing Yukawa interactions. What if they are not diagonal?

i.e.,
$$\mathcal{L}_{Yukawa} = -G^L_{AB} \left[\bar{R}_A (\phi^\dagger L_B) + (\bar{L}_B \phi) R_A \right]$$
$$- G^U_{AB} \left[\bar{U}_A (\phi^{c\dagger} Q_B) + (\bar{Q}_B \phi^c) U_A \right]$$
$$- G^D_{AB} \left[\bar{D}_A (\phi^\dagger Q_B) + (\bar{Q}_B \phi) D_A \right]$$

here, $A \neq B$ gives mixing.

After SSB, get nontrivial mass matrices

$$\mathcal{L}_{mass} = -\frac{1}{\sqrt{2}} G^{L}_{AB}\, a\, \bar{\ell}_{R_A} \ell_{L_B} + h.c.$$
$$-\frac{1}{\sqrt{2}} G^{U}_{AB}\, a\, \bar{u}_{R_A} u_{L_B} + h.c.$$
$$-\frac{1}{\sqrt{2}} G^{D}_{AB}\, a\, \bar{d}_{R_A} d_{L_B} + h.c.$$

Physical mass operators can be found by diagonalizing as usual. Need 3x3 unitary matrix for each component of the field ↳ 3 generations

Denote mass eigenstates w/ hat

Define 7 3x3 matrices to achieve this,

$$\ell_{L_A} = (U^{\ell}_{L})_{AB}\, \hat{\ell}_{L_B} \qquad u_{L_A} = (U^{u}_{L})_{AB}\, \hat{u}_{L_B}$$
$$\ell_{R_A} = (U^{\ell}_{R})_{AB}\, \hat{\ell}_{R_B} \qquad u_{R_A} = (U^{u}_{R})_{AB}\, \hat{u}_{R_B}$$
$$\nu_{L_A} = (U^{\nu}_{L})_{AB}\, \hat{\nu}_{L_B} \qquad d_{L_A} = (U^{d}_{L})_{AB}\, \hat{d}_{L_B}$$
$$d_{R_A} = (U^{d}_{R})_{AB}\, \hat{d}_{R_B}$$

Notice that there are no possible mass terms for the neutrinos in this theory. Therefore, we can choose any $(U_L^\nu)_{AB}$.

Let's choose $(U_L^\nu)_{AB} = (U_L^\ell)_{AB}$,

so that
$$L_A = \begin{pmatrix} \nu_{LA} \\ \ell_{LA} \end{pmatrix} = (U_L^\ell)_{AB} \begin{pmatrix} \hat{\nu}_{LB} \\ \hat{\ell}_{LB} \end{pmatrix}$$
$$= (U_L^\ell)_{AB}\, \hat{L}_B$$

However, for the quark doublet Q, we have

$$Q_{LA} = \begin{pmatrix} u_{LA} \\ d_{LA} \end{pmatrix} = \begin{pmatrix} (U_L^u)_{AB}\, \hat{u}_{LB} \\ (U_L^d)_{AB}\, \hat{d}_{LB} \end{pmatrix}$$
$$= (U_L^u)_{AB} \begin{pmatrix} \hat{u}_{LB} \\ \hat{d}'_{LB} \end{pmatrix}$$

where,
$$\hat{d}'_{LA} = \left[(U_L^u)^\dagger (U_L^d) \right]_{AB} \hat{d}_{LB}$$
$$\equiv V_{AB}\, \hat{d}_{AB}$$

↳ Cabibbo-Kobayashi-Maskawa (CKM) matrix

The CKM matrix is a 3×3 matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Suppressing A, B generation indices we have

$$L = U_L^{\ell}\, \hat{L} \qquad R = U_R^{\ell}\, \hat{R}$$

$$Q = U_L^{u}\, \hat{Q}' \qquad U = U_R^{u}\, \hat{U}$$

$$D = U_R^{d}\, \hat{D}$$

Substitute this into $\mathcal{L}_{EW}$ to express it in terms of mass eigenstates,

$$\mathcal{L}_{EW} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^{a} W^{\mu\nu a}$$

$$+ (D_\mu \phi)^{\dagger} (D^\mu \phi) + \mu^2 \phi^{\dagger}\phi - \frac{\lambda}{3!} (\phi^{\dagger}\phi)^2$$

$$+ \frac{i}{2} \bar{\hat{L}} \not{D} \hat{L} + \frac{i}{2} \bar{\hat{R}} \not{D} \hat{R} + h.c.$$

$$+ \frac{i}{2} \bar{\hat{Q}}' \not{D} \hat{Q}' + \frac{i}{2} \bar{\hat{U}} \not{D} \hat{U} + \frac{i}{2} \bar{\hat{D}} \not{D} \hat{D} + h.c.$$

$$- \Big[\bar{\hat{R}} (U_R^{\ell\dagger} G^{L} U_L^{\ell}) \phi^{\dagger} \hat{L} + h.c.$$

$$+ \bar{\hat{U}} (U_R^{u\dagger} G^{U} U_L^{u}) \phi^{c\dagger} \hat{Q}' + h.c.$$

$$+ \bar{\hat{D}} (U_R^{d\dagger} G^{D} U_L^{d}) \phi^{\dagger} \hat{Q}' + h.c. \Big]$$

In the mass basis, see that propagating states are $\hat{Q}'$, not $\hat{Q}$. Let's see what this does.

For the Yukawa terms, choose $U^{\ell}_{L/R}$, $U^{u}_{L/R}$, $U^{d}_{L/R}$ to diagonalize the Yukawa couplings!

$$\Rightarrow \quad (U_R^{\ell\dagger} G^L U_L^{\ell})_{AB} \equiv \delta_{AB} \tilde{G}^L_A$$
$$(U_R^{u\dagger} G^U U_L^{u})_{AB} \equiv \delta_{AB} \tilde{G}^U_A$$
$$(U_R^{d\dagger} G^D U_L^{d})_{AB} \equiv \delta_{AB} \tilde{G}^D_A$$

Still have only 9 Yukawa parameters.

Notice that the mass terms are the same as before,

e.g.;
$$\mathcal{L}_{mass} \supset -\frac{1}{\sqrt{2}} \tilde{G}^D_A \left[\bar{\hat{d}}_{RA} (0, v) \begin{pmatrix} \hat{u}_{AL} \\ \hat{d}'_{AL} \end{pmatrix} + (\bar{\hat{u}}_{LA}, \bar{\hat{d}}'_{LA}) \begin{pmatrix} 0 \\ v \end{pmatrix} \hat{d}_{RA} \right]$$
$$= -\frac{1}{\sqrt{2}} \tilde{G}^D_A v \left(\bar{\hat{d}}_{RA} \hat{d}'_{LA} + \bar{\hat{d}}'_{LA} \hat{d}_{RA} \right)$$
$$= -\frac{1}{\sqrt{2}} \tilde{G}_A v \bar{\hat{d}}_A \hat{d}_A$$

$$\Rightarrow \quad m_A = \frac{1}{\sqrt{2}} \tilde{G}_A a \quad , \text{ as before.}$$

The kinetic terms are

$$\frac{i}{2}\bar{\hat{Q}}'_L \not{D} \hat{Q}'_L = \frac{i}{2}(\bar{\hat{u}}_L, \bar{\hat{d}}_L V^\dagger) \not{D} \begin{pmatrix} \hat{u}_L \\ V\hat{d}_L \end{pmatrix}$$

with,

$$D_\mu = \left(\begin{array}{c|c} \partial_\mu + \frac{1}{2} i g W^3_\mu + \frac{1}{6} i g' B_\mu & +\frac{1}{2} i g \sqrt{2} W^+_\mu \\ \hline +\frac{1}{2} i g \sqrt{2} W^-_\mu & \partial_\mu - \frac{1}{2} i g W^3_\mu + \frac{1}{6} i g' B_\mu \end{array}\right)$$

For $\bar{\hat{u}}\hat{u}$ & $\bar{\hat{d}}\hat{d}$ terms, no effect as $V^\dagger V = \mathbb{1}$.

Therefore, there is no flavor changing neutral currents (FCNC)

So, interaction w/ Z^0-boson is as before,

$$\mathcal{L}_{NC} = -ie J^\mu_{em} A_\mu - \frac{ig}{\cos\theta_W} J^\mu_Z Z_\mu$$

with $\boxed{J^\mu_{em} = \sum_f Q_f \bar{f} \gamma^\mu f}$

$$\boxed{J^\mu_Z = \sum_f \bar{f} \gamma^\mu (v_f - a_f \gamma_5) f}$$

	Q_f	v_f	a_f
ν	0	$1/4$	$1/4$
ℓ	-1	$-1/4 + \sin^2\theta_W$	$-1/4$
u	$+2/3$	$1/4 - \frac{2}{3}\sin^2\theta_W$	$1/4$
d	$-1/3$	$-1/4 + \frac{1}{3}\sin^2\theta_W$	$-1/4$

Off-diagonal terms, however, change flavor via $W_\mu^\pm$ exchange.

$$\mathcal{L}_{cc} = \bar{u}_{A_L} \gamma^\mu \frac{1}{\sqrt{2}} i g W_\mu^+ V_{AB} d_{B_L} + h.c.$$

$$= \frac{ig}{\sqrt{2}} \bar{u}_A \gamma^\mu P_L V_{AB} d_{B_L} + h.c.$$

$\hookrightarrow$ flavor changing!

So, mass eigenstates = propagating states are not the flavor eigenstates. The CKM matrix contains more parameters (absorbed from the general Yukawa matrices). How many more independent parameters are there?

A 3×3 unitary matrix has 9 real parameters
If the entries are not complex $\Rightarrow$ unitary $\rightarrow$ orthogonal.
But, 3×3 orthogonal matrices have 3 real parameters.
$\Rightarrow$ expect 6 phases for CKM.
However, not all phases are observable. Each quark field can absorb a phase (6), but there remains an overall global phase.
$\Rightarrow$ CKM has 4 real parameters (3 magnitude, 1 phase)

This phase can be shown to be CP violating.

It is useful to parametrize the CKM using the 3 magnitudes & 1 phase called δ (9 components of V_{AB}, $A = u, c, t$; $B = d, s, b$).

The <u>Standard Parametrization</u>: $\theta_{12}, \theta_{13}, \theta_{23}, \delta$

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $c_{jk} = \cos\theta_{jk}$, $s_{jk} = \sin\theta_{jk}$.

$$\theta_{12} \simeq 13.04^\circ (5^\circ)$$

$$\theta_{13} = 0.201^\circ (11^\circ) \qquad \delta = 68.8^\circ (4.5^\circ)$$

$$\theta_{23} = 2.38^\circ (6^\circ)$$

this gives
$$|V_{CKM}| \sim \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ 0.22 & 0.98 & 0.04 \\ 0.009 & 0.04 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$\lambda \sim 0.2$

The unitarity of the CKM offers interesting insight into quark physics.

$$V^{\dagger}V = \mathbb{1} \Rightarrow (V^{\dagger})_{lk} (V)_{kj} = \delta_{lj}$$

or, $\boxed{V^*_{kl} V_{kj} = \delta_{jl}}$ 9 conditions!

Each case $j \neq l$ is an equation w/ 3 complex numbers adding to zero. Can thus represent as a triangle in complex plane.

Pick $j = d$, $l = b$, $k = u, c, t$

$$\Rightarrow \quad V_{ud} V^*_{ub} + V_{cd} V^*_{cb} + V_{td} V^*_{tb} = 0$$

complex numbers

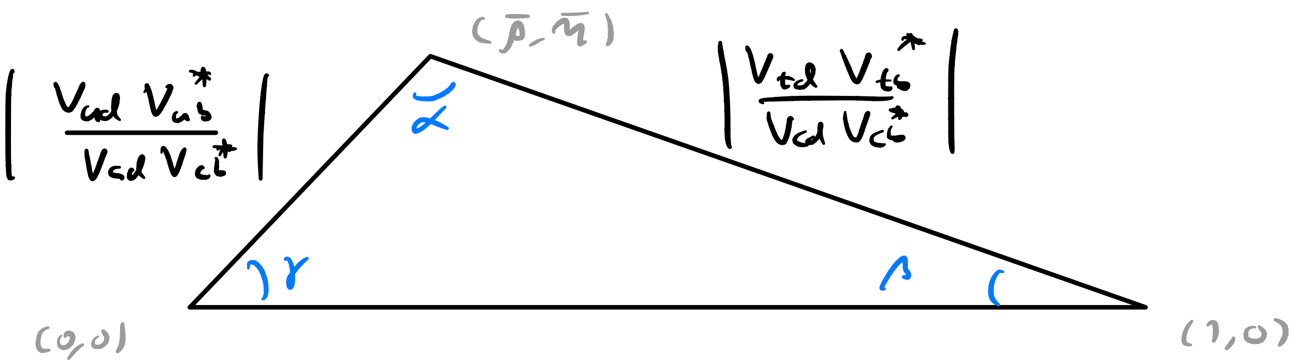
$\Rightarrow$ vector in $\mathbb{C}$-plane

triangle

To get convenient normalization, divide by $V_{cd} V^*_{cb}$

$$\Rightarrow \quad 1 + \frac{V_{td} V^*_{tb}}{V_{cd} V^*_{cb}} = - \frac{V_{ud} V^*_{ub}}{V_{cd} V^*_{cb}} \equiv \bar{\rho} + i\bar{\eta}$$

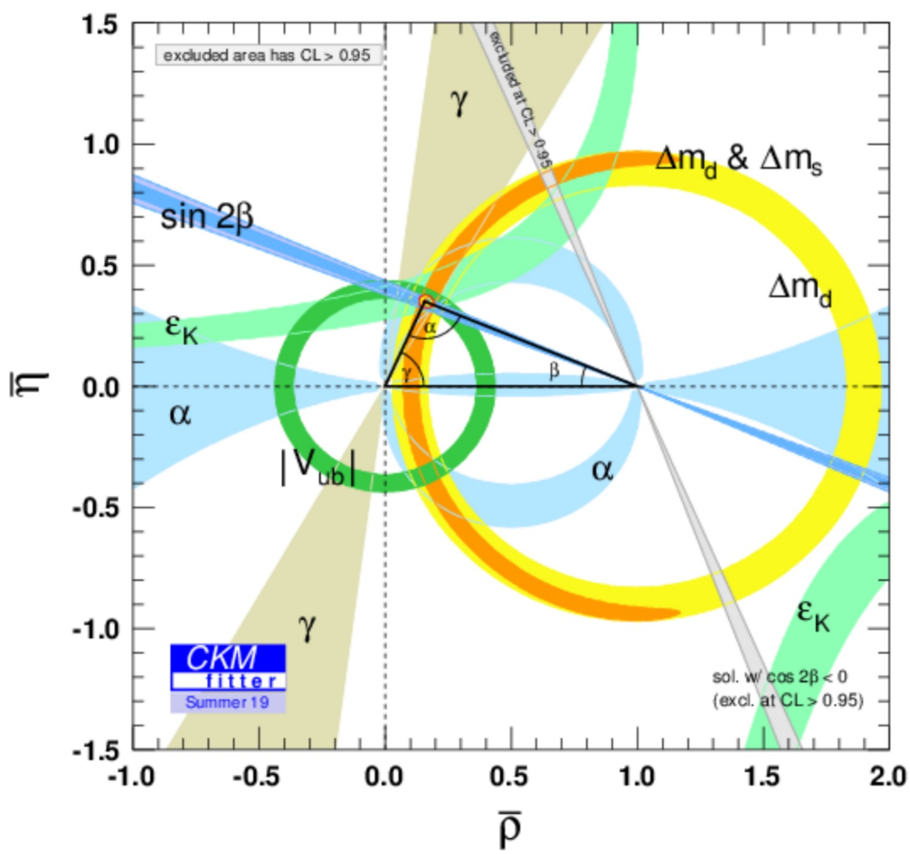
(Wolfenstein parameters)



With 3 generations of quarks, 3×3 V is unitary.

If there are 4 (or more) generations, then 3×3 <u>NOT</u> unitary.

$\Rightarrow$ Checking closure of triangle is a check on the 6 quark model of the SM.



Minimal Standard Model

To get minimal SM, must add QCD.

$\Rightarrow$ Make each quark a triplet under $SU(3)_c$,

& add $SU(3)_c$ gluons G^a_μ, $a=1,\dots,8$

via covariant derivatives & kinetic term.

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu$$

$\hookrightarrow$ New coupling

$$\Rightarrow \quad D_\mu \to D_\mu^{(EW)} + i g_s G^a_\mu (T_a)_{jk}$$

$\hookrightarrow$ $SU(3)_c$

The complete MSM, before SSB, in flavor basis, is

$$\boxed{\begin{aligned}
\mathcal{L}_{MSM} = & -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} \\
& + \frac{1}{2} i \bar{L}_A \not{D} L_A + \frac{1}{2} i \bar{R}_A \not{D} R_A + h.c. \\
& + \frac{1}{2} i \bar{Q}_A \not{D} Q_A + \frac{1}{2} i \bar{U}_A \not{D} U_A + \frac{1}{2} i \bar{D}_A \not{D} D_A + h.c. \\
& + (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \frac{\lambda}{3!} (\phi^\dagger \phi)^2 \\
& - G^L_{AB} [\bar{R}_A (\phi^\dagger L_A) + (\bar{L}_A \phi) R_A] \\
& - G^U_{AB} [\bar{U}_A (\phi^{c\dagger} Q_B) + (\bar{Q}_B \phi^c) U_A] \\
& - G^D_{AB} [\bar{D}_A (\phi^\dagger Q_B) + (\bar{Q}_B \phi) D_A]
\end{aligned}}$$

Representation content: $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

$$(\underset{\sim}{m}, \underset{\sim}{n})_Y$$

$\underset{\sim}{m}$ → $SU(3)_c$

$\underset{\sim}{n}$ → $SU(2)_L$

Y → weak hypercharge

Vectors

$(\underset{\sim}{8}, \underset{\sim}{1})_0$	$(\underset{\sim}{1}, \underset{\sim}{3})_0$	$(\underset{\sim}{1}, \underset{\sim}{1})_0$
G_μ^a	W_μ^a	B_μ

Fermions

Quarks	$(\underset{\sim}{3}, \underset{\sim}{2})_{\frac{1}{2}}$	$(\underset{\sim}{3}, \underset{\sim}{1})_{4/3}$	$(\underset{\sim}{3}, \underset{\sim}{1})_{-2/3}$
	Q_L	U_R	D_R
Leptons	$(\underset{\sim}{1}, \underset{\sim}{2})_{-1}$	$(\underset{\sim}{1}, \underset{\sim}{1})_2$	
	L	R	

Scalars

$$(\underset{\sim}{1}, \underset{\sim}{2})_1$$

$$\phi$$

There are 18 parameters in the MSM which must be constrained from experiment.

3	gauge couplings		g, g', g_s
9	masses	– leptons	m_e, m_μ, m_τ
		– quarks	$m_u, m_d, m_s, m_c, m_b, m_t$
4	CKM parameters	– 3 angles	$\theta_{12}, \theta_{13}, \theta_{23}$
		– 1 phase	δ
2	Higgs parameters	– mass	m_{H^0}
		– self coupling	λ

There is an additional parameter, θ_{cp}, which comes from an additional term in the Lagrange density which can violate CP in strong interactions.

dual field

$$\mathcal{L}_{\text{strong } cp} = \theta_{cp} \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}$$

This is a topological (nonperturbative) effect, & has no contributions to perturbation theory. However, all current experimental results give $\boxed{\theta_{cp} = 0}$ (strong CP Problem)