**1**. Show that the global U(1) symmetry, $\psi \to e^{i\alpha}\psi$ with $\alpha \in \mathbb{R}$, of the spinor field theory

$$\mathcal{L} = \frac{1}{2} i\bar{\psi}\overleftrightarrow{\partial\!\!\!/}\psi + \text{h.c.} - m\bar{\psi}\psi\,,$$

leads to a conserved current $\mathcal{J}^\mu = \bar{\psi}\gamma^\mu\psi$. Show explicitly that this current is conserved.

**2**. Derive the classical equations of motion for spinor electrodynamics given the Lagrange density

$$\mathcal{L} = \frac{1}{2} i\bar{\psi}\overleftrightarrow{D\!\!\!\!/}\psi + \text{h.c.} - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,$$

with $D_\mu = \partial_\mu + iqA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the Euler-Lagrange equations

$$\partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)}\right) = \frac{\delta\mathcal{L}}{\delta\psi}\,, \qquad \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\bar{\psi})}\right) = \frac{\delta\mathcal{L}}{\delta\bar{\psi}}\,, \qquad \partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu A_\nu)}\right) = \frac{\delta\mathcal{L}}{\delta A_\nu}\,.$$

**3**. An alternative Lagrange density for the classical free electromagnetic field is

$$\mathcal{L}' = -\frac{1}{2}\partial_\mu A_\nu\,\partial^\mu A^\nu\,.$$

(**a**) Under what assumption does $\mathcal{L}'$ yield the free inhomogeneous Maxwell equations?

(**b**) With this assumption, show that $\mathcal{L}'$ differs from $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ by a four-divergence.

**4**. Verify that the field strength tensor $F_{\mu\nu}$ can be computed through the commutator $iqF_{\mu\nu} = [D_\mu, D_\nu]$.

**5**. Show that the radiative transition, $e^- \to e^- + \gamma$, is forbidden in vacuum.

**6**. Consider "Bhabha scattering", $e^-e^+ \to e^-e^+$, within QED in the high-energy limit, i.e., the ultra-relativistic limit $m_e^2/s \to 0$. Compare the experimentally measured unpolarized differential cross-section to the theoretical prediction at leading order in $\alpha = e^2/4\pi$, the fine-structure constant.

(**a**) Working in the center-of-momentum (CM) frame, what are the energies and momenta of each particle in the reaction as a function of the Mandelstam invariant $s$? What is the invariant momentum transfer, $t$, as a function of $s$ and $\cos\theta$ where $\theta$ is the scattering angle?

(**b**) Compute the unpolarized differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$, where $\Omega$ is the solid angle of the electron in the $e^-e^+$ CM frame, to order $\alpha^2$ in terms of the Mandelstam invariants $s$ and $t$.

(**c**) Make a Semi-log plot of the $\mathcal{O}(\alpha^2)$ theoretical $\mathrm{d}\sigma/\mathrm{d}\Omega$ vs. $\cos\theta \in [-.8, .8]$ for each CM energy $\sqrt{s}/\,\mathrm{GeV} = \{14, 22, 34.8, 38.3, 43.6\}$. Plot the cross-section in nb, and restrict the $y$-axis to $(\mathrm{d}\sigma/\mathrm{d}\Omega)/\,\mathrm{nb} \in [0.001, 10.000]$. Plot the experimental data, measured from the TASSO experiment at PETRA, for each of the CM energies over the theoretical curves. Compare and comment on the quality of the theoretical description of the experimental data. **Note:** The data file presents the cross-section as $s \cdot \mathrm{d}\sigma/\mathrm{d}\Omega$. The data file was obtained from HEPData at https://www.hepdata.net/record/ins249557. The article by the TASSO collaboration may be helpful, https://link.springer.com/article/10.1007/BF01579904.

(**d**) Make a plot of the ratio of the experimentally measured differential cross-section to the leading order QED prediction as a function of $\cos\theta \in [-.8, .8]$ for each CM energy $\sqrt{s}/\,\mathrm{GeV} = \{14, 22, 34.8, 38.3, 43.6\}$. Restrict the $y$ axis between 0.5 and 1.5. Compare and comment on the quality of the theoretical description of the experimental data. **Hint:** Plot each energy on a separate plot to see if you notice any subtle trends.