1. **Reading & Research:** It can be shown that the beta function for the weak interaction coupling is negative, similar to QCD. However, the weak interaction does not exhibit any "confinement" phenomena. Give an explanation of why the weak interaction does not have similar physics to QCD.

2. The leading order beta function for QCD gives a running coupling

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0} \frac{1}{\log(Q^2/\Lambda^2_{\text{QCD}})} \,,$$

where $\beta_0 = 11 - 2N_f/3$, where N_f is the number of active quark flavors and Q^2 is the energy scale of the system. Active flavors mean quarks that contribute to the system, that is those with $Q^2 > m_q^2$. The scale Λ_{QCD} is defined where the coupling diverges. Qualitatively, $m_s < \Lambda_{\text{QCD}} < m_c$, where m_s and m_c are the strange and charm quark masses.

   (a) If at a given Q_0^2, $\alpha_s(Q_0^2)$ is measured, write an expression for Λ^2_{QCD} in terms of β_0, Q_0^2, and $\alpha_s(Q_0^2)$.

   (b) Fixing α_s at the Z^0 boson mass, $Q_0 = m_Z$, determine a numerical value for Λ_{QCD} in MeV. **Hint:** Since Λ_{QCD} is between the strange and charm masses, one needs to run the coupling from the Z boson scale to through the heavy quark thresholds. At each quark mass, one must match the coupling above and below this scale, where the number of active quark flavors differ. Perform this matching and running until you find a scale at which the coupling diverges. Use the Review of Particle Physics to find the masses and $\alpha_s(m_Z)$.

3. To first approximation, the electric form factor of the proton is measured to be

$$G_E(Q^2) = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2} \,,$$

where Q^2 is the momentum transfer to the proton and $\Lambda^2 \approx 0.71\,\text{GeV}^2$.

   (a) Determine the root-mean-square charge radius of the proton, $\sqrt{\langle r^2 \rangle}$, in fm. Compare the result to the experimentally measured value in the Review of Particle Physics.

   (b) Compute the distribution of electric charge $\rho(\mathbf{x})$ associated with this form-factor. **Hint:** Assume a static distribution, such that the form-factor is defined as

$$G_E(Q^2) = \int \mathrm{d}^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}}\, \rho(\mathbf{x}) \,,$$

   where $Q^2 = -q^2$ and $q^2 = -|\mathbf{q}|^2$.