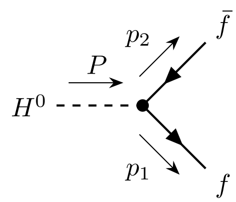
Due Thursday May 9 at 12:00pm

The discovery of the Higgs boson (H^0) in 2012 at the Large Hadron Collider (LHC) at CERN completed the Standard Model of Particle Physics. Here, we will calculate the decay rate of the Higgs to various channels. Use the following parameters: $m_{H^0} = 125.5\,\text{GeV}$, $m_t = 173.5\,\text{GeV}$, $m_b = 4.4\,\text{GeV}$, $m_c = 1.77\,\text{GeV}$, $m_s = 0.093\,\text{GeV}$, $m_\tau = 1.777\,\text{GeV}$, $m_\mu = 0.105\,\text{GeV}$, $m_Z = 91.19\,\text{GeV}$, $m_W = 80.38\,\text{GeV}$, $\alpha_s = 0.1184$, $\alpha_e = 1/127.918$, and $\sin^2\theta_W = 0.2312$. Use the Feynman rules in the unitary gauge.

1. Consider the decay $H^0 \to f\bar{f}$ where the fermions are $f = \{\mu, \tau, s, c, b\}$. Calculate the decay rate $\Gamma(H^0 \to f\bar{f})$ for each fermion at leading order within the Standard Model.

Solution: In the unitary gauge, the decay amplitude of the Higgs to fermion-antifermion pairs is

$i\mathcal{M} =$



$$= -i\frac{g}{2}\frac{m_f}{m_W}\delta_{jk}\bar{u}(p_1)v(p_2)\,,$$

where the δ_{jk} is the color conservation factor for quarks. The spin-averaged matrix element is

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= \frac{g^2}{4}\frac{m_f^2}{m_W^2}\sum_{j,k}(\delta_{jk})^2\sum_{s_1,s_2}[\bar{v}(p_2)u(p_1)][\bar{u}(p_1)v(p_2)]\,,\\ &= \frac{g^2}{4}\frac{m_f^2}{m_W^2}N_c(f)\,\mathrm{tr}((\not{p}_2 - m_f)(\not{p}_1 + m_f))\,,\\ &= g^2\frac{m_f^2}{m_W^2}N_c(f)(p_1\cdot p_2 - m_f^2)\,,\end{aligned}$$

where $\sum_{j,k}(\delta_{jk})^2 = N_c(f)$ is the color factor for the fermion, $N_c(q) = 3$ and $N_c(\ell) = 1$. Now, $p_2 = P - p_1$, so $p_1\cdot p_2 = p_1\cdot P - p_1^2 = p_1\cdot P - m_f^2$. In the rest frame of the Higgs, $p_1\cdot P = E_1 m_{H^0}$. The energy of the fermion in this frame is $E_1 = m_{H^0}/2$, so the spin-averaged matrix element is

$$\langle|\mathcal{M}|^2\rangle = \frac{g^2}{2}\frac{m_f^2}{m_W^2}N_c(f)(m_{H^0}^2 - 4m_f^2)\,.$$

The decay rate in the Higgs rest frame is

$$\mathrm{d}\Gamma = \frac{1}{32\pi^2}\langle|\mathcal{M}|^2\rangle\frac{|\mathbf{p}_1|}{m_{H^0}^2}\mathrm{d}\Omega\,.$$

Noting that the decay momentum is $|\mathbf{p}_1| = \sqrt{m_{H^0}^2 - 4m_f^2}/2$, and the matrix element is independent of the solid angle Ω, we have the decay rate given by

$$\Gamma(H^0 \to f\bar{f}) = \frac{g^2}{32\pi}\frac{m_f^2 m_{H^0}}{m_W^2}N_c(f)\left(1 - \frac{4m_f^2}{m_{H^0}^2}\right)^{3/2}\,.$$

The coupling g can be written in terms of e and $\sin\theta_W$ as $g = e/\sin\theta_W$. Furthermore, $e^2 = 4\pi\alpha$, so we write the decay rate as

$$\Gamma(H^0 \to f\bar{f}) = \frac{\alpha}{8\sin^2\theta_W}\frac{m_f^2 m_{H^0}}{m_W^2} N_c(f)\left(1 - \frac{4m_f^2}{m_{H^0}^2}\right)^{3/2} .$$

For each fermion mass, we find the following decay rates

$$\Gamma(H^0 \to b\bar{b}) = 4.73\times 10^{-3}\,\text{GeV}\,,$$

$$\Gamma(H^0 \to c\bar{c}) = 7.71\times 10^{-4}\,\text{GeV}\,,$$

$$\Gamma(H^0 \to s\bar{s}) = 2.13\times 10^{-6}\,\text{GeV}\,,$$

$$\Gamma(H^0 \to \tau\bar{\tau}) = 2.59\times 10^{-4}\,\text{GeV}\,,$$

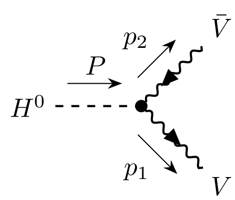
$$\Gamma(H^0 \to \mu\bar{\mu}) = 9.05\times 10^{-7}\,\text{GeV}\,.$$

2. Calculate the $H^0 \to W^+W^-$, and $H^0 \to Z^0Z^0$ decay rates at leading order within the Standard Model, assuming that the decay products are on-mass-shell.

Solution: Since $m_{H^0} < 2m_W$ and $m_{H^0} < 2m_Z$ for on-shell W and Z bosons, then the decay rate is identically zero,

$$\Gamma(H^0 \to W^+W^-) = \Gamma(H^0 \to Z^0Z^0) = 0\,.$$

For a fun exercise, let's consider a situation where the Higgs is heavier than the decay threshold. Both the $H^0 \to W^+W^-$ and $H^0 \to Z^0Z^0$ amplitudes have similar structure. So, let $V = W^+$ or Z^0, from which we have

$i\mathcal{M} =$ 

$$= iG_V m_V\, g_{\mu\nu}\varepsilon^{\mu\,*}(p_1)\varepsilon^{\nu\,*}(p_2)\,,$$

where for $V = W^+$, $G_W = g$ and for $V = Z^0$, $G_Z = g/\cos\theta_W$. Computing the spin-averaged matrix element, we find

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= G_V^2 m_V^2\, g_{\mu\nu}g_{\alpha\beta}\sum_{\text{spins}}\varepsilon^{\mu\,*}(p_1)\varepsilon^{\nu\,*}(p_2)\,\varepsilon^{\alpha}(p_1)\varepsilon^{\beta}(p_2)\,,\\ &= G_V^2 m_V^2\, g^{\mu\nu}g^{\alpha\beta}\left(-g_{\mu\alpha} + \frac{p_{1\,\mu}p_{1\,\alpha}}{m_V^2}\right)\left(-g_{\nu\beta} + \frac{p_{2\,\nu}p_{2\,\beta}}{m_V^2}\right)\,,\end{aligned}$$

where we used the polarization sums for massive vector bosons,

$$\sum_{\text{spin}} \varepsilon^{\mu\,*}(p)\varepsilon^{\nu}(p) = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_V^2}\,.$$

Contracting the Lorentz indices,

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= G_V^2 m_V^2 \left(-g^{\mu\nu} + \frac{p_1^\mu p_1^\nu}{m_V^2}\right)\left(-g_{\mu\nu} + \frac{p_{2\,\mu} p_{2\,\nu}}{m_V^2}\right)\,,\\ &= G_V^2 m_V^2 \left(4 - \frac{p_1^2}{m_V^2} - \frac{p_2^2}{m_V^2} + \frac{(p_1\cdot p_2)^2}{m_V^4}\right)\,.\end{aligned}$$

Noting that $p_1^2 = p_2^2 = m_V^2$, and also that in the Higgs rest frame $p_1 \cdot p_2 = p_1 \cdot P - p_1^2 = E_1 m_{H^0} - m_V^2$. Since $E_1 = m_{H^0}/2$, $p_1 \cdot p_2 = m_{H^0}^2/2 - m_V^2$. Simplifying the spin-averaged matrix element,

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= G_V^2 m_V^2 \left(2 + \frac{(m_{H^0}^2 - 2m_V^2)^2}{4m_V^4}\right)\,,\\ &= G_V^2 m_V^2 \left(3 + \frac{m_{H^0}^4}{4m_V^4} - \frac{m_{H^0}^2}{m_V^2}\right)\,,\\ &= G_V^2\,\frac{m_{H^0}^4}{4m_V^2}\left(1 - \frac{4m_V^2}{m_{H^0}^2} + \frac{3}{4}\left(\frac{4m_V^2}{m_{H^0}^2}\right)^2\right)\,.\end{aligned}$$

Using $|\mathbf{p}_1| = \sqrt{m_{H^0}^2 - 4m_V^2}/2$, the partial width is then computed as

$$\begin{aligned}\Gamma(H^0 \to V\bar{V}) &= \frac{1}{\mathcal{S}_V}\frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_{H^0}^2}\,\langle|\mathcal{M}|^2\rangle\,,\\ &= \frac{1}{\mathcal{S}_V}\frac{G_V^2}{64\pi}\frac{m_{H^0}^3}{m_V^2}\left(1 - \frac{4m_V^2}{m_{H^0}^2} + \frac{3}{4}\left(\frac{4m_V^2}{m_{H^0}^2}\right)^2\right)\sqrt{1 - \frac{4m_V^2}{m_{H^0}^2}}\,,\end{aligned}$$

where $\mathcal{S}_V$ is the symmetry factor for identical particles, $\mathcal{S}_W = 1$ and $\mathcal{S}_Z = 2$. Now, $G_V \propto g = e/\sin\theta_W$, and $e^2 = 4\pi\alpha$. For W^+W^- final states, we have

$$\begin{aligned}\Gamma(H^0 \to W^+W^-) &= \frac{g^2}{64\pi}\frac{m_{H^0}^3}{m_W^2}\left(1 - \frac{4m_W^2}{m_{H^0}^2} + \frac{3}{4}\left(\frac{4m_W^2}{m_{H^0}^2}\right)^2\right)\sqrt{1 - \frac{4m_W^2}{m_{H^0}^2}}\,,\\ &= \frac{\alpha}{16\sin^2\theta_W}\frac{m_{H^0}^3}{m_W^2}\left(1 - \frac{4m_W^2}{m_{H^0}^2} + \frac{3}{4}\left(\frac{4m_W^2}{m_{H^0}^2}\right)^2\right)\sqrt{1 - \frac{4m_W^2}{m_{H^0}^2}}\,.\end{aligned}$$

For Z^0Z^0 final states, the decay rate is

$$\begin{aligned}\Gamma(H^0 \to Z^0Z^0) &= \frac{1}{2}\frac{g^2}{64\pi\cos^2\theta_W}\frac{m_{H^0}^3}{m_Z^2}\left(1-\frac{4m_Z^2}{m_{H^0}^2}+\frac{3}{4}\left(\frac{4m_Z^2}{m_{H^0}^2}\right)^2\right)\sqrt{1-\frac{4m_Z^2}{m_{H^0}^2}}\,, \\ &= \frac{\alpha}{32\sin^2\theta_W\cos^2\theta_W}\frac{m_{H^0}^3}{m_Z^2}\left(1-\frac{4m_Z^2}{m_{H^0}^2}+\frac{3}{4}\left(\frac{4m_Z^2}{m_{H^0}^2}\right)^2\right)\sqrt{1-\frac{4m_Z^2}{m_{H^0}^2}}\,, \\ &= \frac{\alpha}{32\sin^2\theta_W}\frac{m_{H^0}^3}{m_W^2}\left(1-\frac{4m_Z^2}{m_{H^0}^2}+\frac{3}{4}\left(\frac{4m_Z^2}{m_{H^0}^2}\right)^2\right)\sqrt{1-\frac{4m_Z^2}{m_{H^0}^2}}\,,\end{aligned}$$

where we used $m_W = m_Z\cos\theta_W$.

3. Calculate the $H^0 \to \gamma\gamma$ decay rate at leading order in the Standard Model. Note that the Higgs does not couple to photons at tree-level, and thus can only couple to photons via a fermion or W-boson loop. Draw the contributing diagrams. Which fermion loop gives the dominant contribution? The amplitude corresponding to the diagrams is given by

$$\mathcal{M}(H^0 \to \gamma\gamma) = \frac{e^2 g}{(4\pi)^2 m_W}\,F\,(p_1\cdot p_2\, g_{\mu\nu} - p_2^\mu p_1^\nu)\,\varepsilon^{\mu\,*}(p_1)\varepsilon^{\nu\,*}(p_2)\,,$$

where g is the weak coupling, and F includes contributions from both W-boson and fermion loops,

$$F = F_W(\beta_W) + \sum_f N_c\,Q_f^2\,F_f(\beta_f)\,.$$

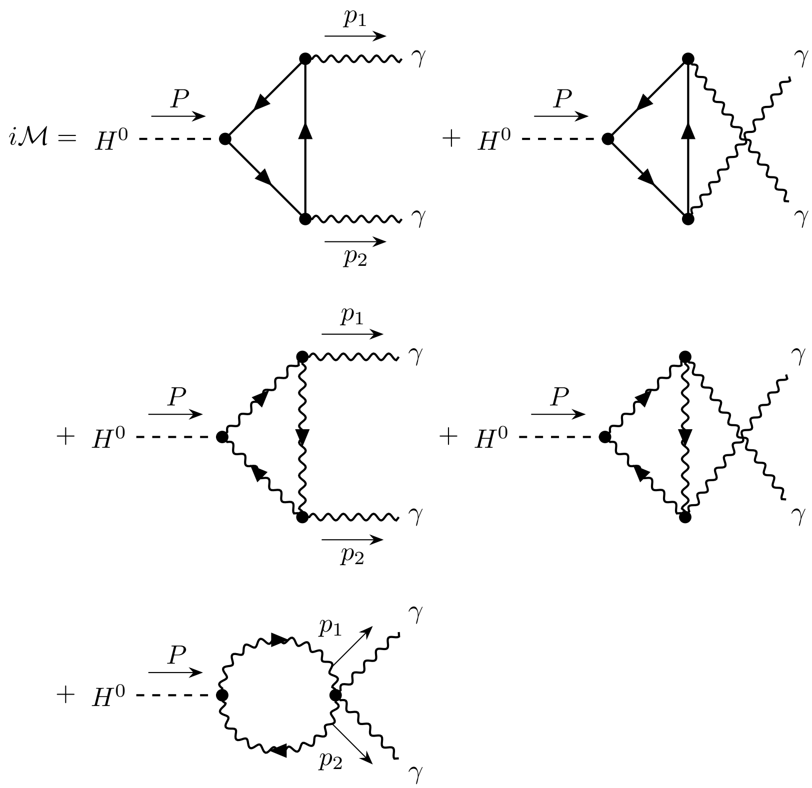
Here N_c is the color factor for the fermions, Q_f the charge of the fermions, and $\beta_W = 4m_W^2/m_H^2$, $\beta_f = 4m_f^2/m_H^2$. The functions F_W and F_f are

$$\begin{aligned}F_W(\beta) &= 2+3\beta+3\beta(2-\beta)\,f(\beta)\,, \\ F_f(\beta) &= -2\beta\left[1+(1-\beta)\,f(\beta)\right]\,,\end{aligned}$$

where

$$f(\beta) = \begin{cases} \arcsin^2(\beta^{-1/2}) & \text{for } \beta \geq 1 \\ -\dfrac{1}{4}\left[\log\left(\dfrac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}\right) - i\pi\right]^2 & \text{for } \beta < 1\,. \end{cases}$$

Solution: From the Feynman rules, we can identify three classes of diagrams



The spin-averaged matrix element is

$$\langle|\mathcal{M}|^2\rangle = \left(\frac{e^2 g}{(4\pi)^2 m_W}\right)^2 |F|^2 (p_1 \cdot p_2 g^{\mu\nu} - p_2^\mu p_1^\nu)(p_1 \cdot p_2 g^{\alpha\beta} - p_2^\alpha p_1^\beta) \sum_{\text{spins}} \varepsilon_\mu^*(p_1)\varepsilon_\nu^*(p_2)\varepsilon_\alpha(p_1)\varepsilon_\beta(p_2)\,.$$

Since the decay products are photons, the polarization sums are simply

$$\sum_{\text{spins}} \varepsilon_\mu^*(p_1)\varepsilon_\alpha(p_1) = -g_{\mu\alpha}\,,$$

and similarly for the second photon. Therefore, the spin-averaged matrix element is

$$\begin{aligned}\langle|\mathcal{M}|^2\rangle &= \left(\frac{e^2 g}{(4\pi)^2 m_W}\right)^2 |F|^2 (p_1 \cdot p_2 g^{\mu\nu} - p_2^\mu p_1^\nu)(p_1 \cdot p_2 g_{\mu\nu} - p_{2\,\mu} p_{1\,\nu})\,,\\ &= 2\left(\frac{e^2 g}{(4\pi)^2 m_W}\right)^2 |F|^2\,(p_1 \cdot p_2)^2\,,\end{aligned}$$

where we used $p_1^2 = p_2^2 = 0$ for photons. The momentum and energy of the photons is $m_{H^0}/2$, thus $p_1 \cdot p_2 = m_{H^0}^2/2$. Therefore, with $e = g\sin\theta_W$ and $e^2 = 4\pi\alpha$, we have

$$\langle|\mathcal{M}|^2\rangle = \frac{\alpha^3}{32\pi\sin^2\theta_W}\frac{m_{H^0}^4}{m_W^2}\,|F|^2\,.$$

The decay rate is then

$$\Gamma(H^0 \to \gamma\gamma) = \frac{1}{2}\frac{1}{8\pi}\frac{|\mathbf{p}_1|}{m_{H^0}^2}\langle|\mathcal{M}|^2\rangle\,,$$
$$= \frac{1}{1024\pi^2}\frac{\alpha^3}{\sin^2\theta_W}\frac{m_{H^0}^3}{m_W^2}|F|^2$$

where the first factor of 1/2 is due to the symmetry factor for identical photons, and we used $|\mathbf{p}_1| = m_{H^0}/2$.
The modulus-square of the form factor is

$$|F|^2 = \left|F_W(\beta_W) + \sum_f N_c\,Q_f^2\,F_f(\beta_f)\right|^2,$$
$$= |F_W(\beta_W)|^2 + \left|\sum_f N_c\,Q_f^2\,F_f(\beta_f)\right|^2 + 2\,\mathrm{Re}\left(\sum_f N_c\,Q_f^2\,F_W^*(\beta_W)F_f(\beta_f)\right).$$

Evaluating the form factors, $F_W(\beta_W) = 8.34 \implies |F_W(\beta_W)|^2 = 69.57$. For the fermion loops, note that $F_f(\beta) \propto \beta$, so for $\beta_f \ll 1$, $F_f(\beta_f) \ll 1$. The neutrinos are massless in the SM, thus give no contribution. The electron, up quark, and down quark are also negligible compared with the Higgs boson mass, $\beta_e, \beta_u, \beta_d \ll 1$, thus we exclude their contribution to the fermion loops. Recalling that $N_c(q) = 3$ and $N_c(\ell) = 1$, we can compute the fermion loop form factor,

$$\sum_f N_c\,Q_f^2\,F_f(\beta_f) = -1.755 - 0.0843i\,,$$

which gives $|\sum_f N_c\,Q_f^2\,F_f(\beta_f)|^2 = 3.087$. To assess the largest fermion contribution, consider the quantity

$$\epsilon_f \equiv \frac{|N_c\,Q_f^2\,F_f(\beta_f)|}{\sum_f |N_c\,Q_f^2\,F_f(\beta_f)|}\,.$$

We find

$$\epsilon_t = 94.01\,\%\,,$$
$$\epsilon_b = 2.18\,\%\,,$$
$$\epsilon_c = 2.16\,\%\,,$$
$$\epsilon_s = 4.01\times10^{-3}\,\%\,,$$
$$\epsilon_\tau = 1.63\,\%\,,$$
$$\epsilon_\mu = 1.49\times10^{-2}\,\%\,,$$

which indicates that the top quark is the dominant contribution to the fermion loop.

Finally, the interference term yields $2\,\mathrm{Re}\,(\sum_f N_c\,Q_f^2\,F_W^*(\beta_W)F_f(\beta_f)) = -29.28$. Therefore, the total $|F|^2$ is

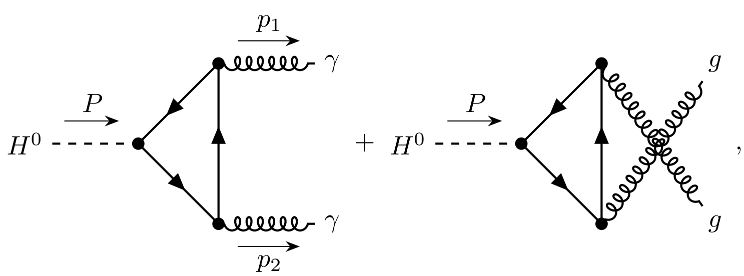
$$|F| = 69.57 + 3.087 - 29.28\,,$$
$$= 43.38\,.$$

Combining this with the decay rate formula, we find

$$\Gamma(H^0 \to \gamma\gamma) = 2.71 \times 10^{-6}\,\text{GeV}$$

4. Calculate the $H^0 \to gg$ decay rate to leading order in the Standard Model. Note that the Higgs does not couple to gluons at tree-level, and thus can only couple to gluons via a quark loop. Which quark loops give the dominant contribution? The difference between the amplitudes for gluon and photon final states is simply $eQ_f \to g_s$ and a color factor.

Solution: From the Feynman rules, the only contributing diagrams involve quark loops,

$i\mathcal{M} =$ 

$$= \frac{g_s^2 g}{(4\pi)^2 m_W} \sum_q F_q(\beta_q)\,(p_1 \cdot p_2\,g_{\mu\nu} - p_2^\mu p_1^\nu)\,\varepsilon^{\mu\,*}(p_1)\varepsilon^{\nu\,*}(p_2)\,,$$

where we identified the contributions from the $H^0 \to \gamma\gamma$ associated with these diagrams, and substituting $e^2 N_c(f) Q_f^2 \to g_s^2$ and only summing over quarks. The decay rate is then

$$\Gamma(H^0 \to gg) = \frac{1}{1024\pi^2}\frac{\alpha\alpha_s^2}{\sin^2\theta_W}\frac{m_{H^0}^3}{m_W^2}\left|\sum_q F_q(\beta_q)\right|^2.$$

Numerically, we find

$$\Gamma(H^0 \to gg) = 2.36 \times 10^{-6}\,\text{GeV}\,.$$

The dominant contribution, like for $H^0 \to \gamma\gamma$, is the top quark loop.

5. Calculate the total Higgs decay width and the branching ratio to the various modes. You should find that the branching ratio to the two-photon decay mode is small, however the Higgs was discovered in this channel. Comment on the possible reasons why the $H^0 \to \gamma\gamma$ may have been advantageous for its discovery.

Solution: The total decay rate is

$$\Gamma_{H^0} = \Gamma(H^0 \to \gamma\gamma) + \Gamma(H^0 \to gg) + \sum_{f=\{b,c,s,\tau,\mu\}} \Gamma(H^0 \to f\bar{f}) \,,$$
$$= 5.79 \times 10^{-3}\,\text{GeV} \,.$$

The branching ratios are

$$\text{BR}(H^0 \to b\bar{b}) = 81.717\,\% \,,$$
$$\text{BR}(H^0 \to c\bar{c}) = 13.306\,\% \,,$$
$$\text{BR}(H^0 \to s\bar{s}) = 0.037\,\% \,,$$
$$\text{BR}(H^0 \to \tau\bar{\tau}) = 4.470\,\% \,,$$
$$\text{BR}(H^0 \to \mu\bar{\mu}) = 0.016\,\% \,,$$
$$\text{BR}(H^0 \to gg) = 0.408\,\% \,,$$
$$\text{BR}(H^0 \to \gamma\gamma) = 0.047\,\% \,.$$

While the branching ratio for $H^0 \to \gamma\gamma$ is small, the signal is not contaminated by the large background of hadronic modes found in the quark branching ratios, which at the LHC is the dominant background.

6. Compare the calculated branching ratios to those observed in experiment. Refer to the *Review of Particle Physics* for a summery of the observed decay modes.

Solution: The observed experimental branching ratios for this set of decay modes are

$$\text{BR}(H^0 \to b\bar{b}) \approx 53\,\% \,,$$
$$\text{BR}(H^0 \to \tau\bar{\tau}) \approx 6\,\% \,,$$
$$\text{BR}(H^0 \to \mu\bar{\mu}) \approx 2.6 \times 10^{-2}\,\% \,,$$
$$\text{BR}(H^0 \to \gamma\gamma) \approx 0.25\,\% \,.$$

We note that although the relative ratios are similar to the theoretical values, there are many other observed decay modes which we do not consider, including the three-body mode $H^0 \to WW^*$, which has a sizable branching fraction at around 25 %.