Feynman Rules - Yukawa theory

The Lagrangian density for a Yukawa theory of a spinor field and real scalar field is given by

$$\mathcal{L} = \frac{i}{2}\bar{\psi}\overleftrightarrow{\partial\!\!\!/}\psi + \text{h.c.} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - M\bar{\psi}\psi - \frac{1}{2}m^2\varphi^2 - g\varphi\bar{\psi}\Gamma\psi$$

where M is the mass of the fermion, m is the mass of the boson, and g is the boson-fermion coupling. Here Γ can be either I (the 4×4 identity) or γ^5, depending on the parity of the scalar field φ. In the following α and β are the spinor indices.

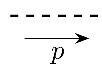
Feynman Rules

Here we give the Feynman rules for the scattering amplitude $\mathcal{M}$,

$$i\mathcal{M} = \text{sum of all connected, amputated diagrams}\,,$$

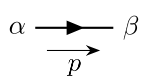
where the diagrams are evaluated according to the following rules:

- Draw all topologically distinct diagrams at a given order;
- For each internal scalar line, attach a propagator



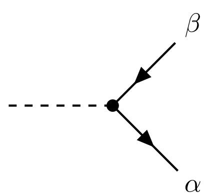
$$= \frac{i}{p^2 - m^2 + i\epsilon}\,;$$

- For each internal spinor line, attach a propagator



$$= \frac{i(\not{p} + M)_{\alpha\beta}}{p^2 - M^2 + i\epsilon}\,;$$

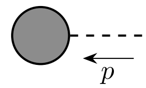
- For each vertex, assign



$$= -ig\,\Gamma_{\beta\alpha}\ \text{(either } \Gamma_{\beta\alpha} = \delta_{\beta\alpha} \text{ or } \gamma^5_{\beta\alpha}\text{)};$$

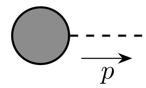
- For each external line, place the particle on the mass-shell $p^2 = m^2$ and attach a wavefunction factor

“incoming scalar”



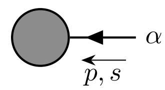
$= 1\,;$

“outgoing scalar”



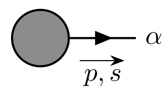
$= 1\,;$

“incoming fermion”



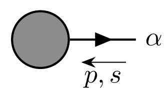
$= u_\alpha(p, s)\,;$

“outgoing fermion”



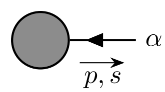
$= \bar{u}_\alpha(p, s)\,,$

“incoming anti-fermion”



$= \bar{v}_\alpha(p, s)\,;$

“outgoing anti-fermion”



$= v_\alpha(p, s)\,;$

- Impose momentum conservation at each vertex;
- For each internal loop momentum k not fixed by momentum conservation, integrate $\displaystyle\int \frac{\mathrm{d}^4 k}{(2\pi)^4}$;
- For each fermion loop, multiply the diagram by (-1);
- For each set of diagram which are only distinguished by interchanging two external fermion lines, multiply one of the diagrams by (-1);