WILLIAM & MARY
CHARTERED 1693

PHYS 772 – The Standard Model of Particle Physics

Final Exam

Due: Tuesday, May 06 at 4:00pm

Term: Spring 2025

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The discovery of the Higgs boson (H^0) in 2012 at the Large Hadron Collider (LHC) at CERN completed the Standard Model of Particle Physics. Here, we will calculate the decay rate of the Higgs to various channels. Use the following parameters: $m_{H^0} = 125.5\,\text{GeV}$, $m_t = 173.5\,\text{GeV}$, $m_b = 4.4\,\text{GeV}$, $m_c = 1.77\,\text{GeV}$, $m_s = 0.093\,\text{GeV}$, $m_\tau = 1.777\,\text{GeV}$, $m_\mu = 0.105\,\text{GeV}$, $m_Z = 91.19\,\text{GeV}$, $m_W = 80.38\,\text{GeV}$, $\alpha_s = 0.1184$, $\alpha_e = 1/127.918$, and $\sin^2\theta_W = 0.2312$. Use the Feynman rules in the unitarity gauge.

1. Consider the decay $H^0 \to f\bar{f}$ where the fermions are $f = \{\mu, \tau, s, c, b\}$. Calculate the decay rate $\Gamma(H^0 \to f\bar{f})$ for each fermion at leading order within the Standard Model.

2. Calculate the $H^0 \to W^+W^-$, and $H^0 \to Z^0Z^0$ decay rates at leading order within the Standard Model, assuming that the decay products are on-mass-shell.

3. Calculate the $H^0 \to \gamma\gamma$ decay rate at leading order in the Standard Model. Note that the Higgs does not couple to photons at tree-level, and thus can only couple to photons via a fermion or W-boson loop. Draw the contributing diagrams. Which fermion loop gives the dominant contribution? The amplitude corresponding to the diagrams is given by

$$\mathcal{M}(H^0 \to \gamma\gamma) = \frac{e^2 g}{(4\pi)^2 m_W}\, F\,(p_1 \cdot p_2\, g_{\mu\nu} - p_2^\mu p_1^\nu)\,\varepsilon^{\mu\,*}(p_1)\varepsilon^{\nu\,*}(p_2)\,,$$

   where g is the weak coupling, and F includes contributions from both W-boson and fermion loops,

$$F = F_W(\beta_W) + \sum_f N_c\, Q_f^2\, F_f(\beta_f)\,.$$

   Here N_c is the color factor for the fermions, Q_f the charge of the fermions, and $\beta_W = 4m_W^2/m_H^2$, $\beta_f = 4m_f^2/m_H^2$. The functions F_W and F_f are

$$F_W(\beta) = 2 + 3\beta + 3\beta(2-\beta)\,f(\beta)\,,$$
$$F_f(\beta) = -2\beta\left[1 + (1-\beta)\,f(\beta)\right]\,,$$

   where

$$f(\beta) = \begin{cases} \arcsin^2(\beta^{-1/2}) & \text{for } \beta \geq 1 \\ -\dfrac{1}{4}\left[\log\left(\dfrac{1+\sqrt{1-\beta}}{1-\sqrt{1-\beta}}\right) - i\pi\right]^2 & \text{for } \beta < 1\,. \end{cases}$$

4. Calculate the $H^0 \to gg$ decay rate to leading order in the Standard Model. Note that the Higgs does not couple to gluons at tree-level, and thus can only couple to gluons via a quark loop. Which quark loops give the dominant contribution? The difference between the amplitudes for gluon and photon final states is simply $eQ_f \to g_s$ and a color factor.

5. Calculate the total Higgs decay width and the branching ratio to the various modes from the results of parts 1-5. You should find that the branching ratio to the two-photon decay mode is small, however the Higgs was discovered in this channel. Comment on the possible reasons why the $H^0 \to \gamma\gamma$ may have been advantageous for its discovery.

6. Compare the calculated branching ratios to those observed in experiment. Refer to the *Review of Particle Physics* for a summery of the observed decay modes.