



PHYS 772 – The Standard Model of Particle Physics

Problem Set 1

Due: Tuesday, February 4 at 12:00pm

Term: Spring 2025

Instructor: Andrew W. Jackura

1. Including anti-particles, how many fundamental particles are described within the Standard Model?
2. In units with $\hbar = c = 1$, show that $1 \text{ kg} \approx 5.61 \times 10^{26} \text{ GeV}$, $1 \text{ m} \approx 5.07 \times 10^{15} \text{ GeV}^{-1}$, and $1 \text{ s} \approx 1.52 \times 10^{24} \text{ GeV}^{-1}$.
3. Starting from SI units and setting $\hbar = c = \epsilon_0 = k = 1$, complete Table 1 using only powers of GeV as your units and keeping only three significant figures (k is Boltzmann's constant).
4. Starting from SI units and setting $\hbar = c = \epsilon_0 = k = 1$, complete Table 2 using only powers of GeV as your units and keeping only three significant figures.
5. Using the current *Review of Particle Physics* particle listings or the summary tables (Particle Data Group, <https://pdg.lbl.gov>), answer the following questions. Quote uncertainties for all data where it is given.
 - (a) What are the valence quarks of the D^+ meson? What is its spin-parity (J^P) quantum numbers? What is its mass in MeV? What is its lifetime τ in seconds? Convert the lifetime to a decay width Γ and quote your answer in MeV ($\Gamma = \hbar/\tau$). What is the significance of the quantity $c\tau = 309.8 \mu\text{m}$ quoted in the PDG listings for experiments?
 - (b) What are the branching ratios $\text{BR}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$ and $\text{BR}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$? Note that the branching ratio is defined as the ratio $\text{BR}(X \rightarrow R_1 R_2 \dots) \equiv \Gamma(X \rightarrow R_1 R_2 \dots)/\Gamma$, where $\Gamma(X \rightarrow R_1 R_2 \dots)$ is the *partial decay width* and Γ is the total decay width. What are the partial decay widths $\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)$ and $\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma)$ in MeV?
 - (c) What are the mass and width of the top quark? Does it decay via the strong, weak, or electromagnetic interactions? Why are there no listings for mesons and baryons containing top quarks?
 - (d) What are the valence quarks of the Λ^0 and Λ_c^+ baryons? What are their spins and parities? What are the branching ratios for their semileptonic decay modes into muons and electrons? Can you explain why these branching fractions are very similar for Λ_c^+ and very different for Λ^0 ?
 - (e) What is the branching ratio for the Higgs boson decay into two photons? Estimate the partial decay width $\Gamma(H^0 \rightarrow \gamma\gamma)$. What is the significance of the quantity $P = 62\,625 \text{ MeV}/c$ quoted in the PDG listing? The Higgs was first discovered in this channel at the Large Hadron Collider, even though its branching ratio is very small. Can you explain why this was so?

Table 1: Physical quantities in SI and in natural units.

Physical Quantity	SI Unit	Natural Unit	True Dimension
mass	kg	$5.61 \times 10^{26} \text{ GeV}$	$c^{-2} \cdot \text{GeV}$
length	m	$5.07 \times 10^{15} \text{ GeV}^{-1}$	$\hbar c \cdot \text{GeV}^{-1}$
time	s	$1.52 \times 10^{24} \text{ GeV}^{-1}$	$\hbar \cdot \text{GeV}^{-1}$
charge	C		
temperature	K		
frequency	Hz		
force	N		
pressure	Pa		
angular momentum	$\text{kg} \cdot \text{m}^2/\text{s}$		
energy	J		
power	W		
scalar potential	V		
electric field	V/m		
current	A		
vector potential	$\text{T} \cdot \text{m}$		
magnetic induction	T		
magnetic flux	Wb		
resistance	Ω		
capacitance	F		
inductance	H		

Table 2: Physical constants in SI and in natural units.

Physical Quantity	Value (SI Units)	Value (Natural Units)	True Dimension
Planck constant, \hbar			
speed of light, c			
permittivity, ϵ_0			
Boltzmann constant, k			
Planck constant, h			
Newton constant, G_N			
$m_{\text{Planck}} = \sqrt{\hbar c / G_N}$			
$l_{\text{Planck}} = \sqrt{\hbar G_N / c^3}$			
$t_{\text{Planck}} = \sqrt{\hbar G_N / c^5}$			
$\rho_{\text{Planck}} = c^5 / \hbar G_N^2$			
permeability, μ_0			
electron charge, e			
fine structure constant, α			
electron mass, m_e			
proton mass, m_p			
Bohr radius, a_0			
Rydberg, Ry			
Bohr magneton, μ_e			
nuclear magneton, μ_p			
Stefan constant, σ			
Fermi constant, $G_F / \hbar^3 c^3$	$3.68 \times 10^{48} \text{ kg}^{-2}$	$1.17 \times 10^{-5} \text{ GeV}^{-2}$	GeV^{-2}
Higgs condensate, $v = \sqrt{(\hbar c)^3 / \sqrt{2} G_F}$	$3.94 \times 10^{-8} \text{ J}$	246 GeV	GeV