WILLIAM & MARY
CHARTERED 1693

PHYS 772 – The Standard Model of Particle Physics

Problem Set 6

Due: Tuesday, March 25 at 4:00pm

Term: Spring 2025

Instructor: Andrew W. Jackura

1. Given the generators X^j for a Lie algebra $[X^j, X^k] = c_{jkl}X^l$, normalized such that $\mathrm{tr}(X^jX^k) = \mu_r\delta_{jk}$, show that the structure constants can be computed with

$$c_{jkl} = \frac{1}{\mu_r}\,\mathrm{tr}([X^j, X^k]X^l)\,.$$

   Show that c_{jkl} are antisymmetric under interchange of any two indices.

2. Compute the non-zero structure constants f_{abc} for the $\mathfrak{su}(3)$ algebra $[\lambda_a, \lambda_b] = 2if_{abc}\lambda_c$, where λ_a are the Gell-Mann matrices. **Hint:** It is convenient to use a symbolic algebra software like `Mathematica`.

3. The Gell-Mann matrices also satisfy the relation

$$\{\lambda_a, \lambda_b\} = \frac{4}{3}\delta_{ab}I_3 + 2d_{abc}\lambda_c\,,$$

   where d_{abc} are symmetric under the interchange of any two indices. Compute the non-zero values of d_{abc}. **Hint:** It is convenient to use a symbolic algebra software like `Mathematica`.

4. Show that the $\mathbf{3}^*$ of $\mathfrak{su}(3)$ is inequivalent to the $\mathbf{3}$ of $\mathfrak{su}(3)$. **Hint:** Show that $(-\lambda_a^*)$ cannot be transformed to λ_a by a unitary transformation for every $a = 1, 2, \ldots, 8$.

5. Perform the Clebsch-Gordan decomposition for the following $\mathfrak{su}(3)$ products using Young Tableau, labeling the dimension of each representation: (a) $\mathbf{3}\times\mathbf{3}\times\mathbf{8}$, and (b) $\mathbf{3}\times\mathbf{3}^*\times\mathbf{8}$.

6. Using the current *Review of Particle Physics* particle listings or the summary tables (Particle Data Group, https://pdg.lbl.gov), complete Table 1 for some typical light and strange *mesons*. For hadrons without an explicit charge index, label all possible charges in the multiplet.

7. Using the current *Review of Particle Physics* particle listings or the summary tables (Particle Data Group, https://pdg.lbl.gov), complete Table 2 for some typical light and strange *baryons*. Note that for some listings, the decay width is reported as $\Gamma = -2\,\mathrm{Im}\,(\text{pole position})$. For hadrons without an explicit charge index, label all possible charges in the multiplet.

Table 1: Light and Strange Mesons.

Meson	**Quark Content**	$J^{P(C)}$	$I^{(G)}$	**Charge**	**Mass** / MeV	**Lifetime** / s	**Principle Decay Modes**
$\pi^{\pm}$	$u\bar{d}, d\bar{u}$	0^-	1^-	± 1	139.57	2.60×10^{-8}	$\mu^+\bar{\nu}$
π^0	$u\bar{u} - d\bar{d}$	0^{-+}	1^-	0	134.98		
$K^{\pm}$							
$K^0, \bar{K}^0$							
K_S							
K_L							
η							
η'							
$\rho(770)$							
$\omega(782)$							
$K^*(892)$							
$f_0(500)$							
$f_0(1370)$							
$a_0(980)$							
$a_1(1260)$							
$a_2(1320)$							
$\pi_1(1600)$							

Table 2: Light and Strange Baryons.

Baryon	**Quark Content**	J^P	I	**Charge**	**Mass / MeV**	**Lifetime / s**	**Principle Decay Modes**
p	uud	$1/2^+$	1/2	+1	938.27	stable	—
n							
Λ^0							
$\Sigma^\pm$							
Σ^0							
Ξ^-							
Ξ^0							
$\Delta^{++}(1231)$							
$\Delta^{\pm}(1231)$							
$\Delta^{0}(1231)$							
$\Sigma(1385)$							
$\Xi(1530)$							
Ω^-							
$N(1440)$							
$\Lambda(1405)$							