WILLIAM & MARY
CHARTERED 1693

PHYS 772 – The Standard Model of Particle Physics

Problem Set 7

Due: Tuesday, April 01 at 4:00pm

Term: Spring 2025

Instructor: Andrew W. Jackura

1. Classify the following observed reactions into strong, electromagnetic, and weak processes:

(a) $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$,

(b) $\gamma + p \to \pi^+ + n$,

(c) $p + \bar{p} \to \pi^+ + \pi^- + \pi^0$,

(d) $D^- \to K^+ + 2\pi^-$,

(e) $\Lambda^0 + p \to K^- + 2p$,

(f) $\pi^- + p \to n + e^+ + e^-$.

2. Both the ρ^0 meson and the ω meson are vector mesons, $J^{PC} = 1^{--}$. However, the ρ^0 is observed to strongly decay predominately into 2π, while the ω is observed to decay into 3π. Why is this so?

3. Consider πN scattering at the $\Delta(1232)$ resonance, i.e., at center-of-momentum energies $\sqrt{s} \sim 1232\,\mathrm{MeV}$. For this reaction, $\pi N \to \Delta(1232) \to \pi N$, focus on the following three processes:

(a) $\pi^+ p \to \pi^+ p$ *elastic scattering* via the Δ^{++} resonance,

(b) $\pi^- p \to \pi^- p$ *elastic scattering* via the Δ^0 resonance,

(c) $\pi^- p \to \pi^0 n$ *charge exchange* via the Δ^0 resonance.

Estimate the relative cross sections $\sigma_a : \sigma_b : \sigma_c$.

4. Consider a $q\bar{q}$ meson within an exact flavor SU(3) quark model, i.e., $q = u, d, s$. Assume the meson is flavor neutral. A generic wave function for this meson is given by

$$|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} = \sum_{m_L, m_S} \langle L m_L; S m_S | J m_J \rangle \sum_{s,\bar{s}} \left\langle \frac{1}{2} s; \frac{1}{2} \bar{s} | S m_S \right\rangle$$
$$\times \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\, \varphi_{n,L}(p)\, Y_{L m_L}(\hat{\mathbf{p}})\, |q_s(\mathbf{p}) \bar{q}_{\bar{s}}(-\mathbf{p})\rangle \ ,$$

where n is the radial quantum number, S is the total intrinsic spin, L is the orbital angular momentum, J is the total angular momentum, m_J is the total angular momentum projection on some fixed z-axis, m_L is the orbital angular momentum projection, m_S is the total intrinsic spin projection, $\varphi_{n,L}$ is the momentum-space radial wave function, and Y_{Lm_L} are spherical harmonics. The quarks are spin-1/2 fermions with spin s and $\bar{s}$ for the q and $\bar{q}$, respectively. The two-quark state is defined in the center-of-momentum frame as the usual direct product $|q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle \equiv |q_s(\mathbf{p})\rangle \otimes |\bar{q}_{\bar{s}}(-\mathbf{p})\rangle$.

(a) Determine the allowed values of S.

(b) Show that under parity $\mathcal{P}$, the $q\bar{q}$ meson has an eigenvalue

$$\mathcal{P}\left|n\,^{2S+1}L_J, m_J\right\rangle_{q\bar{q}} = (-1)^{L+1}\left|n\,^{2S+1}L_J, m_J\right\rangle_{q\bar{q}} .$$

Hint: Recall that $\mathcal{P}\left|q_s(\mathbf{p})\right\rangle = \eta_q\left|q_s(-\mathbf{p})\right\rangle$ and $\eta_{\bar{q}} \equiv -\eta_q$.

(c) Show that under charge conjugation $\mathcal{C}$, the $q\bar{q}$ meson has an eigenvalue

$$\mathcal{C}\left|n\,^{2S+1}L_J, m_J\right\rangle_{q\bar{q}} = (-1)^{L+S}\left|n\,^{2S+1}L_J, m_J\right\rangle_{q\bar{q}} .$$

Hint: Recall that $\mathcal{C}\left|q_s(\mathbf{p})\right\rangle = \left|\bar{q}_s(\mathbf{p})\right\rangle$, and under interchange $\mathrm{P}_{12}\left|q_1 q_2\right\rangle = -\left|q_2 q_1\right\rangle$.

(d) Determine *all* allowed J^{PC} quantum numbers for of the meson for $L \leq 3$. List all J^{PC} that are forbidden for $J \leq 3$ (observed mesons with these quantum numbers are called *exotic*, as they are not allowed in the quark model).

(e) List *one* example (if one exist) of an observed unflavored meson for each J^{PC} supermultiplet by searching the Particle Data Group database (https://pdglive.lbl.gov) for *light unflavored mesons*. Are there any examples of observed mesons with exotic quantum numbers?

5. Given the plot of the πN total cross-sections shown in Fig. 1, identify potential resonances and estimate their mass and decay widths, as well as their charge, strange, and baryon quantum numbers. Further, identity their potential spin and isospin quantum numbers. Referring to the *Review of Particle Physics*,

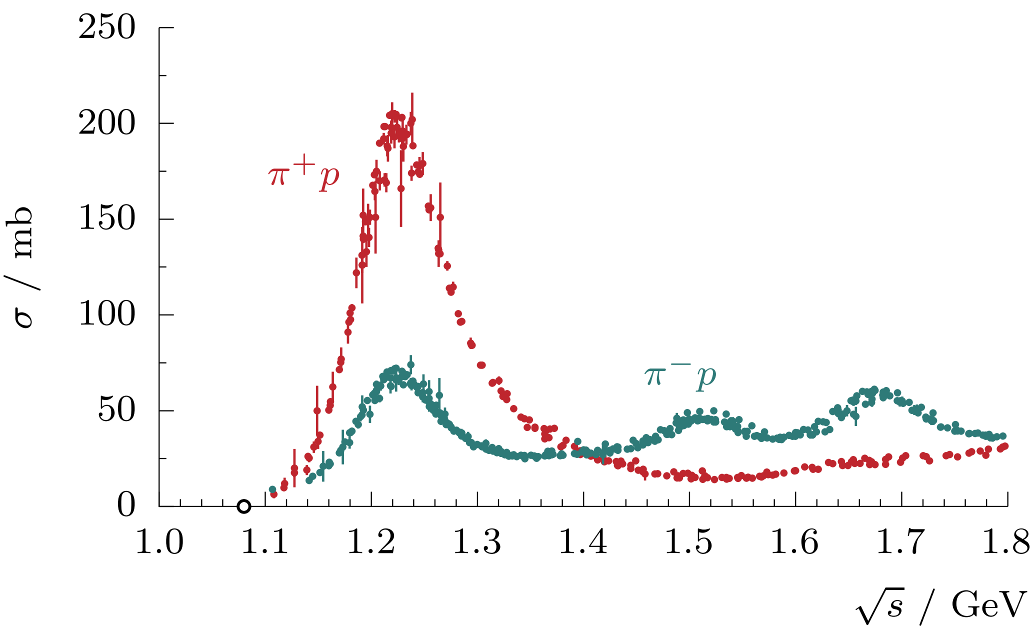

Figure 1: Total πN cross-sections as a function of center-of-momentum frame energy $\sqrt{s}$. Data taken from the *Review of Particle Physics* by the Particle Data Group.