

## PHYS 772 – The Standard Model of Particle Physics

### Problem Set 9

**Due:** Tuesday, April 15 at 4:00pm

**Term:** Spring 2025

**Instructor:** Andrew W. Jackura

**1. Reading & Research:** It can be shown that the beta function for the weak interaction coupling is negative, similar to QCD. However, the weak interaction does not exhibit any “confinement” phenomena. Give a qualitative explanation of why the weak interaction does not have similar physics to QCD.

**2.** The leading order beta function for QCD gives a running coupling

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0} \frac{1}{\log(Q^2/\Lambda_{\text{QCD}}^2)},$$

where  $\beta_0 = 11 - 2N_f/3$ , where  $N_f$  is the number of active quark flavors and  $Q^2$  is the energy scale of the system. Active flavors mean quarks that contribute to the system, that is those with  $Q^2 > m_q^2$ . The scale  $\Lambda_{\text{QCD}}$  is defined where the coupling diverges. Qualitatively,  $m_s < \Lambda_{\text{QCD}} < m_c$ , where  $m_s$  and  $m_c$  are the strange and charm quark masses.

- (a) If at a given  $Q_0^2$ ,  $\alpha_s(Q_0^2)$  is measured, write an expression for  $\Lambda_{\text{QCD}}^2$  in terms of  $\beta_0$ ,  $Q_0^2$ , and  $\alpha_s(Q_0^2)$ .
- (b) Fixing  $\alpha_s$  at the  $Z^0$  boson mass,  $Q_0 = m_Z$ , determine a numerical value for  $\Lambda_{\text{QCD}}$  in MeV. **Hint:** Since  $\Lambda_{\text{QCD}}$  is between the strange and charm masses, one needs to run the coupling from the  $Z$  boson scale to through the heavy quark thresholds. At each quark mass, one must match the coupling above and below this scale, where the number of active quark flavors differ. Perform this matching and running until you find a scale at which the coupling diverges. Use the Review of Particle Physics to find the masses and  $\alpha_s(m_Z)$ .

**3.** A measurement of the electric form factor of the proton yields the following model,

$$G_E(Q^2) = \left(1 + \frac{Q^2}{\Lambda^2}\right)^{-2},$$

where  $Q^2$  is the momentum transfer to the proton and is determined to be  $\Lambda^2 \approx 0.71 \text{ GeV}^2$ .

- (a) Determine the root-mean-square charge radius of the proton,  $\sqrt{\langle r^2 \rangle}$ , in fm. Compare the result to the experimentally measured value in the Review of Particle Physics.
- (b) Compute the distribution of electric charge  $\rho(\mathbf{x})$  associated with this form-factor. **Hint:** Assume a static distribution, such that the form-factor is defined as

$$G_E(Q^2) = \int d^3\mathbf{x} e^{i\mathbf{q}\cdot\mathbf{x}} \rho(\mathbf{x}),$$

where  $Q^2 = -q^2$  and  $q^2 = -|\mathbf{q}|^2$ .