William & Mary
CHARTERED 1693

PHYS 772 – The Standard Model of Particle Physics

Problem Set 7 – Solution

Due: Tuesday, April 01 at 4:00pm

Term: Spring 2025

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1. Classify the following observed reactions into strong, electromagnetic, and weak processes:

   (a) $\pi^- \to \pi^0 + e^- + \bar{\nu}_e$,

   Solution: The presence of the leptons in the final state indicates a non-strong process. Looking at isospin I_3 conservation,

   $$\begin{aligned} & \pi^- \to \pi^0 + e^- + \bar{\nu}_e \,, \\ I_3 : \quad & -1 \to 0 + 0 + 0 \,, \end{aligned}$$

   we see that since $\Delta I_3 = 0 - (-1) \neq 0$, the interaction must be *weak* process.

   (b) $\gamma + p \to \pi^+ + n$,

   Solution: The photo-production of a a pion on a nuclear target is not a pure strong process. Again, isospin I_3 conservation again gives

   $$\begin{aligned} & \gamma + p \to \pi^+ + n \,, \\ I_3 : \quad & 0 + \frac{1}{2} \to 1 + \left(-\frac{1}{2}\right) \,. \end{aligned}$$

   We see that $\Delta I_3 = 0$, and we conclude that this is an *electromagnetic* process.

   (c) $p + \bar{p} \to \pi^+ + \pi^- + \pi^0$,

   Solution: Without the presence of leptons, we suspect that this is a strong processes. As before, isospin I_3 conservation. yields

   $$\begin{aligned} & p + \bar{p} \to \pi^+ + \pi^- + \pi^0 \,, \\ I_3 : \quad & \frac{1}{2} + \left(-\frac{1}{2}\right) \to 1 + (-1) + 0 \,, \end{aligned}$$

   and we see that $\Delta I_3 = 0$. Moreover, the total I is conserved as $p + \bar{p}$ gives either $I = 0$ or 1, and three pions can have $I = 0, 1, 2, 3$. Therefore, this reaction can occur through either $I = 0$ or 1 modes. We conclude that this reaction is a *strong* process.

   (d) $D^- \to K^+ + 2\pi^-$,

Solution: Checking if isospin I_3 is conserved,

$$D^{-1} \to K^+ + \pi^- + \pi^- \,,$$
$$I_3 : \quad \left(-\frac{1}{2}\right) \to \frac{1}{2} + (-1) + (-1)\,,$$

we see that $\Delta I_3 = -3/2 - 1/2 \neq 0$, therefore this decay must be a *weak* process. Another indication that this is a weak process is to look at ΔC and ΔS, which are both non-zero in this reaction, indicating flavor changing which is mediated by the weak interaction.

(e) $\Lambda^0 + p \to K^- + 2p$,

Solution: With no leptons in the reactants or products, and isospin I_3 being conserved $\Delta I_3 = 0$,

$$\Lambda^0 + p \to K^- + p + p\,,$$
$$I_3 : \quad 0 + \frac{1}{2} \to \left(-\frac{1}{2}\right) + \frac{1}{2} + \frac{1}{2}\,,$$

as well as strangeness being conserved $\Delta S = 0$, we find that this process is flavor preserving, and conclude that this reaction is a *strong* process.

(f) $\pi^- + p \to n + e^+ + e^-$.

Solution: Total isospin must be violated due to the production of leptons. Checking if isospin I_3 is conserved,

$$\pi^- + p \to n + e^+ + e^- \,,$$
$$I_3 : \quad (-1) + 1/2 \to (-1/2) + 0 + 0\,,$$

we see that indeed $\Delta I_3 = 0$, so this reaction is not mediated by the weak interaction. Therefore this reaction is an *electromagnetic* process.

2. Both the ρ^0 meson and the ω meson are vector mesons, $J^{PC} = 1^{--}$. However, the ρ^0 is observed to strongly decay predominately into 2π, while the ω is observed to decay into 3π. Why this is so?

Solution: While both hadrons are vector mesons with $J^{PC} = 1^{--}$, note that the ρ^0 is an isovector $I^G = 1^+$ while the ω^0 is an isoscalar $I^G = 0^-$, which can be seen from the Review of Particle Physics. The G-parity of an n-pion state is $G_{n\pi} = (-1)^n$ since $G_\pi = -$. So, if isospin is exact, $\omega^0 \to 3\pi$ is only allowed since $G_\omega = -$, while $\rho^0 \to 2\pi$ is allowed since $G_\rho = +$. Since isospin is broken, but mildly, this means that these decay modes are dominant.

3. Consider πN scattering at the $\Delta(1232)$ resonance, i.e., at center-of-momentum energies $\sqrt{s} \sim 1232\,\text{MeV}$. For this reaction, $\pi N \to \Delta(1232) \to \pi N$, focus on the following three processes:

(a) $\pi^+ p \to \pi^+ p$ *elastic scattering* via the Δ^{++} resonance,

(b) $\pi^- p \to \pi^- p$ *elastic scattering* via the Δ^0 resonance,

(c) $\pi^- p \to \pi^0 n$ *charge exchange* via the Δ^0 resonance.

Estimate the relative cross sections $\sigma_a : \sigma_b : \sigma_c$.

Solution: Since $m_{\pi^\pm} \approx m_{\pi^0}$ $m_p \approx m_n$, and $m_{\Delta^{++}} \approx m_{\Delta^0}$, the approximate isospin symmetry can be considered a good symmetry for this reaction. Let us therefore assume m_π as the mass of the $I = 1$ pion multiplet, m_N as the mass of the $I = 1/2$ nucleon doublet, and m_Δ as the mass of the $I = 3/2$ delta multiplet.

The cross-section in the Δ-region has the structure

$$\sigma_\Delta \propto |\langle f|\, T_\Delta \,|i\rangle|^2 \times (\text{kinematic factors})\,,$$

where T_Δ is the T matrix with Δ quantum numbers, and $|i\rangle$ and $|f\rangle$ are the initial and final states of the processes of interest. Since a particular state can be expressed in terms of the isospin states $|II_3\rangle$, where $I = I_3 = 3/2$ for the Δ^{++} channel and $I = 3/2$, $I_3 = -1/2$ for the Δ^0 channel, then the ratio of the cross sections will involve only ratios of the isospin Clebsch-Gordan coefficients associated with overlaps of either $|\Delta(3/2, 3/2)\rangle$ or $|\Delta(3/2, -1/2)\rangle$. In terms of πN isospin states, the processes are given by

$$|\pi^+ p\rangle = |\pi(1, +1)\rangle \otimes |N(1/2, +1/2)\rangle = |\pi N(3/2, +3/2)\rangle\ ,$$

$$|\pi^- p\rangle = |\pi(1, -1)\rangle \otimes |N(1/2, +1/2)\rangle = \sqrt{\frac{1}{3}}\,|\pi N(3/2, -1/2)\rangle - \sqrt{\frac{2}{3}}\,|\pi N(1/2, -1/2)\rangle\ ,$$

$$|\pi^0 n\rangle = |\pi(1, 0)\rangle \otimes |N(1/2, -1/2)\rangle = \sqrt{\frac{2}{3}}\,|\pi N(3/2, -1/2)\rangle + \sqrt{\frac{1}{3}}\,|\pi N(1/2, -1/2)\rangle\ .$$

Therefore, the ratios of the cross-sections are

$$\begin{aligned}\sigma_a : \sigma_b : \sigma_c &= \left|1 \cdot 1\right|^2 : \left|\sqrt{\frac{1}{3}} \cdot \sqrt{\frac{1}{3}}\right|^2 : \left|\sqrt{\frac{1}{3}} \cdot \sqrt{\frac{2}{3}}\right|^2 ,\\ &= 1 : \frac{1}{9} : \frac{2}{9}\,,\\ &= 9 : 1 : 2\,,\end{aligned}$$

in qualitative agreement with experiment.

4. Consider a $q\bar{q}$ meson within an exact flavor SU(3) quark model, i.e., $q = u, d, s$. Assume the meson is flavor neutral. A generic wave function for this meson is given by

$$|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} = \sum_{m_L, m_S} \langle Lm_L; Sm_S|Jm_J\rangle \sum_{s,\bar{s}} \Big\langle \frac{1}{2}s; \frac{1}{2}\bar{s}|Sm_S\Big\rangle \times \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\, \varphi_{n,L}(p)\, Y_{Lm_L}(\hat{\mathbf{p}})\, |q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle\ ,$$

where n is the radial quantum number, S is the total intrinsic spin, L is the orbital angular momentum, J is the total angular momentum, m_J is the total angular momentum projection on some fixed z-axis, m_L is the orbital angular momentum projection, m_S is the total intrinsic spin projection, $\varphi_{n,L}$ is the momentum-space radial wave function, and Y_{Lm_L} are spherical harmonics. The quarks are spin-1/2 fermions with spin s and $\bar{s}$ for the q and $\bar{q}$, respectively. The two-quark state is defined in the center-of-momentum frame as the usual direct product $|q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle \equiv |q_s(\mathbf{p})\rangle \otimes |\bar{q}_{\bar{s}}(-\mathbf{p})\rangle$.

(a) Determine the allowed values of S.

> **Solution:** Since we have two spin-1/2 objects, the total spin is either $S = 0$ or 1. This can be seen from the Clebsch-Gordan decomposition. If $\mathbf{2}$ is the fundamental representation of $\mathfrak{su}(2)$, then $\mathbf{2} \times \mathbf{2} = \mathbf{1} + \mathbf{3}$. Therefore, we have either a singlet ($S = 0$) or a triplet ($S = 1$) state.

(b) Show that under parity $\mathcal{P}$, the $q\bar{q}$ meson has an eigenvalue

$$\mathcal{P}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} = (-1)^{L+1}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}}\ .$$

Hint: Recall that $\mathcal{P}\,|q_s(\mathbf{p})\rangle = \eta_q\,|q_s(-\mathbf{p})\rangle$ and $\eta_{\bar{q}} \equiv -\eta_q$.

> **Solution:** By direct evaluation,
>
> $$\begin{aligned}
\mathcal{P}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} &= \sum_{m_L,m_S}\langle Lm_L; Sm_S|Jm_J\rangle \sum_{s,\bar{s}}\Big\langle \frac{1}{2}s; \frac{1}{2}\bar{s}\Big|Sm_S\Big\rangle \\
&\quad\times \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(\hat{\mathbf{p}})\mathcal{P}\,|q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle\ , \\
&= \sum_{m_L,m_S}\langle Lm_L; Sm_S|Jm_J\rangle \sum_{s,\bar{s}}\Big\langle \frac{1}{2}s; \frac{1}{2}\bar{s}\Big|Sm_S\Big\rangle \\
&\quad\times \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(\hat{\mathbf{p}})\eta_q\eta_{\bar{q}}\,|q_s(-\mathbf{p})\bar{q}_{\bar{s}}(\mathbf{p})\rangle\ , \\
&= \eta_q\eta_{\bar{q}}\sum_{m_L,m_S}\langle Lm_L; Sm_S|Jm_J\rangle \sum_{s,\bar{s}}\Big\langle \frac{1}{2}s; \frac{1}{2}\bar{s}\Big|Sm_S\Big\rangle \\
&\quad\times \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(-\hat{\mathbf{p}})\,|q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle\ , \\
&= \eta_q\eta_{\bar{q}}(-1)^L\sum_{m_L,m_S}\langle Lm_L; Sm_S|Jm_J\rangle \sum_{s,\bar{s}}\Big\langle \frac{1}{2}s; \frac{1}{2}\bar{s}\Big|Sm_S\Big\rangle \\
&\quad\times \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(\hat{\mathbf{p}})\,|q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle\ , \\
&= \eta_q\eta_{\bar{q}}(-1)^L\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}}\ ,
\end{aligned}$$
>
> where in the third line we let $\mathbf{p} \to -\mathbf{p}$ in the integrand, and then in the fourth line we used $Y_{Lm_L}(-\hat{\mathbf{p}}) = (-1)^L Y_{Lm_L}(\hat{\mathbf{p}})$. Therefore, since $\eta_{\bar{q}} = -\eta_q$, the parity of the quark model

hadron is

$$\mathcal{P}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} = (-1)^{L+1}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}}\ ,$$

since $\eta_q^2 = 1$.

(c) Show that under charge conjugation $\mathcal{C}$, the $q\bar{q}$ meson has an eigenvalue

$$\mathcal{C}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} = (-1)^{L+S}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}}\ .$$

Hint: Recall that $\mathcal{C}\,|q_s(\mathbf{p})\rangle = |\bar{q}_s(\mathbf{p})\rangle$, and under interchange $\mathrm{P}_{12}\,|q_1q_2\rangle = -\,|q_2q_1\rangle$.

Solution: By direct evaluation,

$$\begin{aligned}
\mathcal{C}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} &= \sum_{m_L,m_S}\langle Lm_L;Sm_S|Jm_J\rangle\sum_{s,\bar{s}}\big\langle\tfrac{1}{2}s;\tfrac{1}{2}\bar{s}|Sm_S\big\rangle \\
&\qquad\times\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(\hat{\mathbf{p}})\mathcal{C}\,|q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle\ , \\
&= \sum_{m_L,m_S}\langle Lm_L;Sm_S|Jm_J\rangle\sum_{s,\bar{s}}\big\langle\tfrac{1}{2}s;\tfrac{1}{2}\bar{s}|Sm_S\big\rangle \\
&\qquad\times\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(\hat{\mathbf{p}})\,|\bar{q}_s(\mathbf{p})q_{\bar{s}}(-\mathbf{p})\rangle\ , \\
&= -\sum_{m_L,m_S}\langle Lm_L;Sm_S|Jm_J\rangle\sum_{s,\bar{s}}\big\langle\tfrac{1}{2}s;\tfrac{1}{2}\bar{s}|Sm_S\big\rangle \\
&\qquad\times\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(\hat{\mathbf{p}})\,|q_{\bar{s}}(-\mathbf{p})\bar{q}_s(\mathbf{p})\rangle\ , \\
&= -\sum_{m_L,m_S}\langle Lm_L;Sm_S|Jm_J\rangle\sum_{s,\bar{s}}(-1)^{S+1}\big\langle\tfrac{1}{2}\bar{s};\tfrac{1}{2}s|Sm_S\big\rangle \\
&\qquad\times\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(-\hat{\mathbf{p}})\,|q_{\bar{s}}(\mathbf{p})\bar{q}_s(-\mathbf{p})\rangle\ , \\
&= -(-1)^{S+1}(-1)^L\sum_{m_L,m_S}\langle Lm_L;Sm_S|Jm_J\rangle\sum_{s,\bar{s}}\big\langle\tfrac{1}{2}s;\tfrac{1}{2}\bar{s}|Sm_S\big\rangle \\
&\qquad\times\int\frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\,\varphi_{n,L}(p)\,Y_{Lm_L}(\hat{\mathbf{p}})\,|q_s(\mathbf{p})\bar{q}_{\bar{s}}(-\mathbf{p})\rangle\ , \\
&= (-1)^{L+S}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}}\ ,
\end{aligned}$$

where in the third line we used the antisymmetry properties of fermions, in the fourth line used $\langle j_1m_1;j_2m_2|jm\rangle = (-1)^{j_1+j_2-s}\langle j_2m_2;j_1m_1|jm\rangle$ and the fact that j is integer, and in the fifth line we let $\mathbf{p}\to-\mathbf{p}$ in the integrand, and then used $Y_{Lm_L}(-\hat{\mathbf{p}}) = (-1)^L Y_{Lm_L}(\hat{\mathbf{p}})$.

Therefore, the C-parity of the quark model hadron is

$$\mathcal{C}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}} = (-1)^{L+S}\,|n\,^{2S+1}L_J, m_J\rangle_{q\bar{q}}\ .$$

(d) Determine *all* allowed J^{PC} quantum numbers for of the meson for $L \le 3$. List all J^{PC} that are forbidden for $J \le 3$ (observed mesons with these quantum numbers are called *exotic*, as they are not allowed in the quark model).

Solution: The angular momentum quantum numbers of the $q\bar{q}$ state are $S = 0$ or 1, $L = 0, 1, 2, 3, \ldots$, and $|L - S| \le J \le L + S$. The parity of a given state is $P = (-1)^{L+1}$, and the C-parity is $C = (-1)^{L+S}$. So, we can make a table of the allowed J^{PC} for all $L \le 3$.

Orbital Angular Momentum	**Spin**	J^{PC}
$L = 0$ (S)	$S = 0$	0^{-+}
	$S = 1$	1^{--}
$L = 1$ (P)	$S = 0$	1^{+-}
	$S = 1$	$(0, 1, 2)^{++}$
$L = 2$ (D)	$S = 0$	2^{-+}
	$S = 1$	$(1, 2, 3)^{--}$
$L = 3$ (F)	$S = 0$	3^{+-}
	$S = 1$	$(2, 3, 4)^{++}$

So, the allowed quantum numbers for a $q\bar{q}$ state in the quark model are

$$J^{PC} = (0, 2, \ldots)^{-+}, (1, 3, \ldots)^{+-}, (1, 2, 3, \ldots)^{--}, (0, 1, 2, \ldots)^{++}\ .$$

Notice that there is a set of states not allowed within this model, called exotic, are

$$J^{PC}_{\text{exotic}} = 0^{--}, (1, 3, \ldots)^{-+}, (0, 2, \ldots)^{+-}\ .$$

(e) List *one* example (if one exist) of an observed unflavored meson for each J^{PC} supermultiplet by searching the Particle Data Group database (https://pdglive.lbl.gov) for *light unflavored mesons*. Are there any examples of observed mesons with exotic quantum numbers?

Solution: The following hadrons correspond to the multiplets found in the previous part,

J^{PC}	**hadron**
$(0, 2)^{-+}$	$(\pi^0, \pi_2(1880))$
$(1, 3)^{+-}$	$(b_1(1235), ???)$
$(1, 2, 3)^{--}$	$(\rho(770), ???, \rho_3(1690))$
$(0, 1, 2, 3, 4)^{++}$	$(f_0(500), a_1(1260), f_2(1270), ???, a_4(1970))$

where the "???" indicate that no unflavored neutral hadron has been observed with these quantum numbers.

There has been some observations of exotic quantum numbers, one example being the $\pi_1(1600)$ which has $J^{PC} = 1^{-+}$.

5. Given the plot of the πN total cross-sections shown in Fig. 1, identify potential resonances and estimate their mass and decay widths, as well as their charge, strange, and baryon quantum numbers. Further, identity their potential spin and isospin quantum numbers. Referring to the *Review of Particle Physics*, can you identify candidates for these unstable states?

Solution:
From the total cross-section alone, it is difficult to rigorously identify resonances, one needs to do a *partial wave analysis* on differential cross sections and other angular observables to get a more complete spectroscopic picture. Indeed, in this energy region, for this system, there are about 14 observed resonances in the πN spectrum between threshold and $\sqrt{s} \sim 1.8\,\text{MeV}$, about 5 excited Δ states and 9 N states.

However, a rough estimate may get us some idea of the what the spectral content is of some reaction. Here, we identify four 'strong' bumps, one in $\pi^+ p$, and three in $\pi^- p$. The bump in $\pi^+ p$ peaks around $\sqrt{s} \sim 1.23\,\text{GeV}$, and the bumps in $\pi^- p$ peak at $\sqrt{s} \sim 1.23\,\text{GeV}$, $1.52\,\text{GeV}$, and $1.68\,\text{GeV}$. It is reasonable to assume that the two bumps at $\sqrt{s} \sim 1.23\,\text{MeV}$ are different isospin states of the same resonance. Therefore, we can "easily" identify three resonances, which we call R_1 ($m_1 \sim 1.23\,\text{GeV}$), R_2 ($m_2 \sim 1.52\,\text{GeV}$), and R_3 ($m_3 \sim 1.68\,\text{GeV}$). Assuming a Breit-Wigner form for each resonance,

$$\sigma_R \propto \frac{1}{(s - m_R^2)^2 + m_R^2 \Gamma_R^2}\,,$$

where m_R is the mass of the resonance and Γ_R is the width of the resonance, we that the full-width at half-maximum for the first peak is $\Gamma_{R_1} \sim (1.28 - 1.18)\,\text{GeV} = 0.10\,\text{GeV}$, the second peak is $\Gamma_{R_2} \sim (1.55 - 1.47)\,\text{GeV} = 0.08\,\text{GeV}$, and the third peak is $\Gamma_{R_3} \sim (1.72 - 1.64)\,\text{GeV} = 0.08\,\text{GeV}$. For the higher resonance, we measure with respect to the background cross-section.

Since these states are resonances in $N\pi$, the strangeness for every resonance is $S = 0$ and the baryon number is $B_n = 1$. Since the N is an isospinor, and π is an isovector, the $N\pi$ state can be either $I = 1/2$ or $3/2$. Moreover, since the N is a spin-1/2 object, and the π is spinless, the total spin of the $N\pi$ system is $s = 1/2$. So, the total angular moment J must be $|\ell - 1/2| \leq J \leq \ell + 1/2$, where $\ell = 0, 1, 2, \ldots$ is the orbital angular momentum of the $N\pi$ system.

The isospin quantum numbers for $\pi^+ p$ must be $I = I_3 = 3/2$. Since the first resonance in $\pi^+ p$ must have charge $Q = +2$, we identify this as the Δ^{++}, $R_1 \to \Delta^{++}$, which has a mass $m_{\Delta^{++}} \approx 1.21\,\text{GeV}$ and width $\Gamma_{\Delta^{++}} \approx 0.1\,\text{GeV}$, which agrees with are rough estimate. Since $J^P = 3/2^+$, and the total spin of $N\pi$ is $s = 1/2$, we conclude that the orbital angular momentum of the state is $\ell = 1$, or a P-wave resonance, since the parity of the $N\pi$ state is always $P = (-1)^{\ell+1}$.

For the $\pi^- p$ cross-section, we can have either $I = 1/2$ or $I = 3/2$, which means R_2 and R_3 are either an excited N or Δ state. The charge of R_2 and R_3 is $Q = 0$. Looking at the RPP, we find the following candidates for R_2: $\Delta(1600)$ (with $J^P = 3/2^+$, $m = 1.52\,\text{GeV}$, and $\Gamma = 0.28\,\text{GeV}$), $N(1520)$ (with $J^P = 3/2^-$, $m = 1.51\,\text{GeV}$, and $\Gamma = 0.11\,\text{GeV}$), and $N(1535)$ (with $J^P = 1/2^-$,

$m = 1.51\,\text{GeV}$, and $\Gamma = 0.11\,\text{GeV}$). Given the estimated width is $\Gamma_{R_2} \approx 0.08\,\text{GeV}$, we postulate that R_2 is either $N(1520)$ with $J^P = 3/2^-$ or the $N(1535)$ with $J^P = 1/2^-$. This means that the isospin of R_2 is $I = 1/2$. Since $\ell = J \pm 1/2$, and $P = -1 = (-1)^{\ell+1}$, then for the $J = 1/2$ case we have an S wave state ($\ell = 0$), while for the $J = 3/2$ state it is a D wave reaction ($\ell = 2$).

For the R_3 resonance, the possible states are $N(1650)$ (with $J^P = 1/2^-$, $m = 1.67\,\text{GeV}$, and $\Gamma = 0.14\,\text{GeV}$), $N(1675)$ (with $J^P = 5/2^-$, $m = 1.66\,\text{GeV}$, and $\Gamma = 0.14\,\text{GeV}$), $N(1680)$ (with $J^P = 5/2^+$, $m = 1.67\,\text{GeV}$, and $\Gamma = 0.12\,\text{GeV}$), and $\Delta(1700)$ (with $J^P = 3/2^-$, $m = 1.66\,\text{GeV}$, and $\Gamma = 0.25\,\text{GeV}$). Since the estimated width is $\Gamma_{R_3} \sim 0.08\,\text{GeV}$, we postulate that R_3 is either $N(1650)$ with $J^P = 1/2^-$, $N(1675)$ with $J^P = 5/2^-$, or $N(1680)$ with $J^P = 5/2^+$. Again, the isospin of R_3 is $I = 1/2$. For the $J^P = 1/2^-$ state, the partial wave is $\ell = 0$, for the $J^P = 5/2^-$ state it is $\ell = 2$, and for $J^P = 5/2^+$ it is $\ell = 3$.

To distinguish these states further, one needs to do an angular analysis to determine the spin-parity quantum numbers.

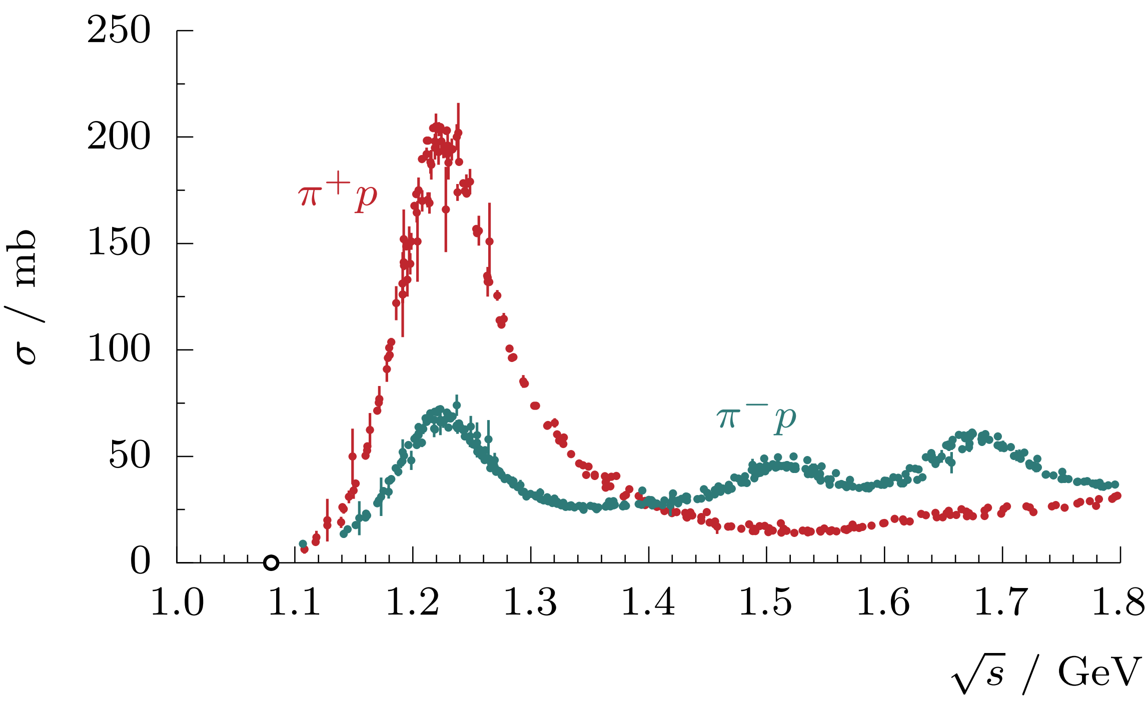

Figure 1: Total πN cross-sections as a function of center-of-momentum frame energy $\sqrt{s}$. Data taken from the *Review of Particle Physics* by the Particle Data Group.